Spring 2017 Econ 164 Final Exam Solution

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June 14, 2017

The exam has 5 questions for 100 points in total. You have 3 hours to finish the exam.

NOTE THAT POINTS VARY BETWEEN QUESTIONS!

DO NOT OPEN THE EXAM BOOK UNTIL THE EXAM BEGINS!

Name:

UID:

Score:

1. Consider the Solow model with neither population growth nor technology progress. Suppose at time 0 the depreciation rate is increased due to global warming. Answer the following questions. (10 points)

a. What is the long run effect on capital per capita of the increase in depreciation rate? Show graphically how you reach the conclusion. (3 points)

Because of the increases in the depreciation rate, the steady state value for capital will decrease.

b. What is the short run effect on capital per capita? Draw the dynamic path of capital per capita after the increase in depreciation rate. (3 points)

The amount of capital will start to decrease, first at a faster rate and then more slowly as we approach the new steady state

c. If we only consider the long run effect, what is the price of global warming? (4 points)

The price of global warming would be the loss of output (and therefore consumption) due to the reduction of the levels of capital

2. The law of motion for aggregate capital in the Solow model is given by

$$K_{t+1} = sA_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta)K_t.$$

Population grows according to $L_{t+1} = (1+n)L_t$ and technology grows according to $A_{t+1} = (1+g)A_t$. Answer the following questions. (15 points)

a. Rewrite the production function in terms of capital, labor, and a labor-augmenting technology factor $\tilde{A}_t$. Find the relationship between $\tilde{A}_t$ and A_t, and the growth rate of $\tilde{A}_t$. (5 points)

The usual production function has to be equal to the one with labor augmenting technology, so

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} = K_t^{\alpha} \left(\tilde{A}_t L_t\right)^{1-\alpha}$$

Thus

$$A_t = \tilde{A}_t^{1-\alpha}$$

And

$$g = (1+\tilde{g})^{1-\alpha} - 1$$

b. Find the law of motion for capital per effective labor $k_t = K_t/(\tilde{A}_t L_t)$. Solve for the steady state of k_t. (5 points)

$$k_{t+1} = \frac{sk_t^{\alpha}}{(1+n)(1+\tilde{g})} + \frac{(1-\delta)\,k_t}{(1+n)(1+\tilde{g})}$$

So

$$k_{ss} = \left(\frac{s}{\delta + n + g + n\tilde{g}}\right)^{1/1-\alpha}$$

c. Find an expression for steady state per capita consumption. What is the growth rate of it? (5 points)

$$\frac{C_t}{L_t} = (1-s)\,\frac{Y_t}{L_t} = (1-s)\left(\frac{s}{\delta + n + g + n\tilde{g}}\right)^{\alpha/1-\alpha} \tilde{A}_t$$

Consumption per capita must grow at the same rate of growth of the labor augmenting technology

3. Government can be a source of low TFP in developing countries. Consider the following problem. There are two sectors in an economy, $s = 1, 2$. They both produce output according to the following technologies:

$$y_1 = A_1^{1-\alpha} l_1^{\alpha}$$

and

$$y_2 = A_2^{1-\alpha} l_2^{\alpha}$$

The output of both sectors are equally valuable as consumption. Thus, total output is the sum of output of the two sectors:

$$Y = y_1 + y_2$$

The resource constraint of this economy is given by

$$l_1 + l_2 = L,$$

where L is the fixed population of this economy. Answer the following questions. (20 points)

a. Find out the efficient allocation of this economy? What is the efficient output? (5 points)

In the efficient allocation

$$MPL_1 = MPL_2$$

So

$$\alpha A_1^{1-\alpha} l_1^{\alpha-1} = \alpha A_2^{1-\alpha} l_2^{\alpha-1} = \alpha A_2^{1-\alpha} (L - l_1)^{\alpha-1}$$

Which means

$$A_1 (L - l_1) = A_2 l_1 \Rightarrow l_1^* = \frac{A_1}{A_1 + A_2} L; \; l_2^* = \frac{A_2}{A_1 + A_2} L$$

And total output is

$$Y = A_1^{1-\alpha}\left(\frac{A_1}{A_1+A_2}L\right)^{\alpha} + A_2^{1-\alpha}\left(\frac{A_2}{A_1+A_2}L\right)^{\alpha} = (A_1+A_2)^{1-\alpha}L^{\alpha}$$

b. Now suppose the government taxes sector 1 for the output they produce and subsides sector 2 for the labor they employ. Sector 1 gets to keep a fraction $(1-\tau)^{1-\alpha} > 0$ of its output and the subsidy rate for each unit of work in sector 2 is $\sigma^{\alpha-1} < 1$. Write down the profit maximization problem for the two firms. (5 points)

Firm one solves

$$\max\,(1-\tau)^{1-\alpha} A_1^{1-\alpha} l_1^{\alpha} - w l_1$$

And firm 2 solves

$$\max A_2^{1-\alpha} l_2^{\alpha} - \sigma^{\alpha-1} w l_2$$

c. Solve for the labor allocation under competitive equilibrium given the tax and subsidy. What is the total output of this economy. (5 points)

Wages have to be the same across the two firms, so

$$\alpha(1-\tau)^{1-\alpha} A_1^{1-\alpha} l_1^{\alpha-1} = \alpha A_2^{1-\alpha} l_2^{\alpha-1}\sigma^{1-\alpha}$$

Thus

$$(1-\tau)A_1(L-l_1) = A_2 l_1 \sigma \Rightarrow l_1 = \frac{(1-\tau)A_1}{(1-\tau)A_1+\sigma A_2}L;\ l_2 = \frac{\sigma A_2}{(1-\tau)A_1+\sigma A_2}L$$

Output is

$$Y = A_1^{1-\alpha}\left(\frac{(1-\tau)A_1}{(1-\tau)A_1+\sigma A_2}L\right)^{\alpha} + A_2^{1-\alpha}\left(\frac{\sigma A_2}{(1-\tau)A_1+\sigma A_2}L\right)^{\alpha} = \frac{A_1(1-\tau)^{\alpha}+\sigma^{\alpha}A_2}{[(1-\tau)A_1+\sigma A_2]^{\alpha}}L^{\alpha}$$

d. The government has to balance its budget such that the tax income from the sector 1 exactly covers the subsidy to sector 2. From that, find a relationship between τ and σ . (5 points)

To balance the budget the tax revenue must equal the subsidy

$$\left(1-(1-\tau)^{1-\alpha}\right)Y_1=\left(1-\sigma^{\alpha-1}\right)wl_2$$

Thus

$$\left(1-(1-\tau)^{1-\alpha}\right)A_1^{1-\alpha}\left(\frac{(1-\tau)A_1}{(1-\tau)A_1+\sigma A_2}L\right)^{\alpha}=\left(1-\sigma^{\alpha-1}\right)\alpha A_2^{1-\alpha}\sigma^{1-\alpha}\left(\frac{\sigma A_2}{(1-\tau)A_1+\sigma A_2}L\right)^{\alpha}$$

Or

$$\left(1-(1-\tau)^{1-\alpha}\right)A_1(1-\tau)^{\alpha}=\left(1-\sigma^{\alpha-1}\right)\alpha A_2\sigma$$

4. An economy has a fixed labor stock L, which is distributed between two sectors of the economy, $s = 1, 2$. Let's say the people in sector 1 own 1/3 of the labor stock, and people in sector 2 own the rest 2/3. Denote the initial labor allocation as L_i, such that $L_1 = 1/3L$ and $L_2 = 2/3L$. The two sectors produce output according to the following technologies:

$$y_1 = A_1 l_1$$

and

$$y_2 = A_2 l_2$$

where l_i is the actual labor used by sector i in production, and we have $A_2 > A_1$. Total output produced is the sum of output of the two sectors.

$$Y = y_1 + y_2$$

The economy's feasibility constraint is given by:

$$L = l_1 + l_2, \text{ } and \text{ } l_1, l_2 \geq 0$$

Answer the following questions. (20 points)

a. What is the efficient allocation of labor? What is the total output when the allocation is efficient? (4 points)

Since $A_2 > A_1$the efficient allocation is

$$l_1 = 0, \text{ } l_2 = L$$

And total output is

$$Y^{eff} = A_2 L$$

b. Suppose there is borrowing constraint to each sector, such that

$$b_i \leq \lambda L_i$$

Show that when $\lambda < \bar{\lambda}$, the competitive economy will not achieve the efficient allocation. Find out $\bar{\lambda}$ given the initial allocation. (10 points)

To achieve the efficient allocation, firm 2 need to borrow $b_2 = {}^1\!/_3 L$. This would not be possible if

$$\frac{1}{3}L < \lambda\frac{2}{3}L$$

Thus $\bar{\lambda} = 1/2$.

c. Find out the equilibrium wage when $\lambda < \bar{\lambda}$. Show that total output is reduced by the presence of the borrowing constraint under this condition. (6 points)

Because firm 1 will stay in business, it must be that

$$w = A_1$$

Firm 2 will borrow

$$b_2 = \lambda\frac{2}{3}L$$

And employ a total of

$$l_2 = \frac{2}{3}L + b_2 = (1+\lambda)\,\frac{2}{3}L$$

Firm 1 employs

$$l_1 = \frac{1}{3}L - b_2 = (1-2\lambda)\,\frac{1}{3}L$$

Total production is

$$Y = A_1\,(1-2\lambda)\,\frac{1}{3}L + A_2\,(1+\lambda)\,\frac{2}{3}L$$

Since a positive amount of labor is used in firm 1, it must be the case that now total output is lower

5. Consider the Schumpeterian model with one-period patent. Output is produced according to:

$$Y_t = A_t L_t,$$

where $L(t)$ is number of production workers and A_t describes the technology available to some or many firms. The number of production workers is fixed at $\bar{L}$.

At period $t-1$, all firms have access to the same technology A_{t-1}. One innovating firm invests in $R\&D$ to achieve a better technology. If $R\&D$ is successful, the innovating firm becomes a leader next period with $A_t = (1+\gamma)A_{t-1}$ while the other firms still have the now inferior technology A_{t-1}. The patent only holds for one period, after that all firms have access to the new technology. If $R\&D$ is not successful, all firms have the old technology.

The probability of successful $R\&D$ depends on the amount of research workers employed,

$$\pi\left(n_{t-1}\right) = \nu \frac{n_{t-1}^{1-\sigma}}{1-\sigma}.$$

where π is the probability of success, and n_{t-1} is the number of research workers employed. The research workers are paid the same wage as the production workers.

Markets are competitive. Firms maximize profits in production and $R\&D$. They discount future profit at rate β. Answer the following questions. (35 points)

a. If $R\&D$ is successful, what is the leaders profit? (5 points)

The profits when R&D is successful are

$$\Pi_t = \left(\gamma A_{t-1}\right)\bar{L}$$

b. Given the profit from successful $R\&D$, how many research workers will be employed? (5 points)

Firm chooses n_{t-1} to solve

$$\max \beta\nu \frac{n_{t-1}^{1-\sigma}}{1-\sigma}\left(\gamma A_{t-1}\right)\bar{L} - A_{t-1}n_{t-1}$$

Thus

$$n_{t-1}^{mkt} = \left(\beta\nu\gamma\bar{L}\right)^{1/\sigma}$$

c. What is the social value of *R&D*? (5 points)

Social Value of R&D is

$$\frac{\beta}{1-\beta\left(1-\bar{\pi}\right)}\nu\frac{n_{t-1}^{1-\sigma}}{1-\sigma}\left(\gamma A_{t-1}\right)\bar{L} - A_{t-1}n_{t-1}$$

d. To maximize the social value of *R&D*, how many workers should be employed? Does the market *R&D* employment fall behind or above the optimal one? (5 points)

The social optimum is

$$n_{t-1}^{social} = \left(\frac{\beta}{1-\beta\left(1-\bar{\pi}\right)}\nu\gamma\bar{L}\right)^{1/\sigma}$$

Since

$$\frac{\beta}{1-\beta\left(1-\bar{\pi}\right)} > \beta$$

The market invests less in R&D than it would be socially optimum.

e. To achieve optimal outcome, the government is to subsidize the innovating firm for employing research workers, what is the subsidy rate? (For this problem, disregard how the government finances the subsidy.) (5 points)

Let's call the subsidy to the wage τ. Then the firm only pays a fraction $1-\tau$ of the wage to the R&D workers. The new firm maximization problem is

$$\max \beta\nu\frac{n_{t-1}^{1-\sigma}}{1-\sigma}\left(\gamma A_{t-1}\right)\bar{L} - A_{t-1}\left(1-\tau\right)n_{t-1}$$

Which gives

$$n_{t-1}^{mkt} = \left[\frac{\beta}{1-\tau}\nu\gamma\bar{L}\right]^{1/\sigma}$$

To make $n_{t-1}^{mkt} = n_{t-1}^{social}$ it must be that

$$\tau = \beta\left(1 - \bar{\pi}\right)$$

f. It is hard for the government to monitor the monitor the firms' activity, do you think subsidizing $R\&D$ is a good idea? If not, what other policies do you propose the government can adopt to boost $R\&D$? (10 points)

No, since the firm would be able to claim the subsidy and not use it in R&D. A different way to boost R&D would be to have patents that last longer than one year.