“Optimal Supply of Public and Private Liquidity”

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Overview

The Paper

- Studies desirability of Public Provision of Safe Assets
- Unlike classic public finance
  - problem is not about when to pay for g
- Paper falls within a new tradition of models:
  - Holmstrom-Tirole
  - Kiyotaki Moore

Features of the model

- Government has limited instruments
  - transfers not agent specific
- Default
  - with dead weight loss
- Pecuniary Externality
  - I affect prices other than through the resource constraint
- Pareto Weights
  - don't coincide with Pareto weights of Competitive Equilibrium

Insights in the Paper

- Default
  - "efficient" market allocation not feasible
- Pecuniary externality
  - produces excessive default
  - dead weight loss
- Government can, if it wanted, implement "efficient" market allocation
  - no need for inefficient private liquidity
- Government doesn't want "efficient" market allocation
  - allow some waste of resources,
  - but more egalitarian

Environment

- Two period model
- Two agents
- Discount factor β
- Identical concave utility
- Endowments:

$$y_0^R = 1 + \Delta, y_1^R = 1$$
$$y_0^P = 1 - \Delta, y_1^P = 1$$

"Efficient" Competitive Allocation

- Presence of a Representative Agent
  - t=0 price of t=1 goods is β
  - no aggregate shocks
  - perfect consumption smoothing:

$$c_0^R = c_1^R$$

- Time Zero Budgets:

$$\begin{aligned} c_0^R + \beta c_0^R &= y_0^R + q y_1^R \\ &= 1 + \Delta + \beta \Longrightarrow \end{aligned}$$

$$\begin{aligned} c_0^R &= 1 + \frac{\Delta}{1+\beta} \Longrightarrow \\ c_0^P &= 1 - \frac{\Delta}{1+\beta}. \end{aligned}$$

- Private Credit:

$$\beta l^p = 1 + \Delta - \left(1 + \frac{\Delta}{1+\beta}\right) = \frac{\beta}{1+\beta}\Delta.$$

Public Liquidity in Competitive Equilibrium

- Add Government
  - Clearing

$$B = \frac{1}{2} b^R + \frac{1}{2} b^p$$

  - Budget Balance

$$-T_0 = qB$$

$$T_1 = B$$

$$\implies -T_0 = \beta T_1$$

- Guess and Verify:

$$c_0^R = c_1^R$$

- Time-Zero Budget

$$c_0^R + \beta c_0^R = y_0^R - T_0 + \beta \left(y_1^R - T_1 \right)$$

$$c_0^R = 1 + \frac{\Delta}{1 + \beta}$$

Ricardian Proposition - No Frictions

- Same Allocation, private liquidity indeterminate
- Maximal Private Liquidity:

$$-T_0 = qb^R = qb^P$$

$$T_1 = b^R$$

  - operation is neutral on each budget
- Minimal Private Liquidity:
  - If $B > \frac{\Delta}{1+\beta}$ then $l^p = 0$
- If $B \leq \frac{\Delta}{1+\beta}$, rich buy all debt $b^R = 2B, b^P = 0$ and private liquidity:

$$l^R = \frac{\Delta}{1+\beta} - B$$

Government's can substitute for private savings

- Add friction back
- Government can save for the poor
- No Private Liquidity
- Set $B = \frac{\Delta}{1+\beta}$
- Then:

$$a^R = l^R = 0$$

  and competitive equilibrium is an equilibrium...
  - result requires no global deviations

Taking Stock

- Without frictions, there's nothing Gov can do!
  - Allocations always the same...
- Even with default
  - Gov can replicate "efficient" competitive allocation
  - but it won't....
- Why would it ever want to deviate from efficiency?
- Negishi's Theorem:
  - Exists Pareto weights that implement competitive allocation
  - BUT, OUR PLANNER HAS DIFFERENT PARETO WEIGHTS
- Point of Azzimonti-Yared
  - deviates from competitive allocation
  - allows inefficient default
  - but more egalitarian solution!

Role of Default / Frictions

- Particular Default
- Default + specuniary externalities
  - standard default, I face an individual price schedule
  - with anonymity, you face a price
- Technical discussion in paper:
  - what prevents from issuing a large amount of debt?
    - In paper, distribution of default cost (and OK!)
  - In general, need collateral
    - borrowing limit inconsistent with anonimity

Version with Collateral + Private Information

- Consider $t = 0$ sales of $t = 1$ endowment
- Endowment can be chopped into continuum of pieces
  - pieces $\omega \in [0, 1]$
  - pieces add to same total endowment

$$y_1^P = \int \lambda(\omega)\, d\omega$$

  - but pieces differ and $\lambda(\omega)$ increasing
- Private information for ω
  - here you sell claim on specific piece
  - Bigio (2015, AER) model with collateral + default (similar)

Endowment Sales with Private Information

- buyer price p^B
  - claim on unit of consumption
- sale price p^S:
  - sell collateral under anonymity
- first-order condition of rich:

$$p^B u'\left(c_0^R\right) = \beta u'\left(c_1^R\right)$$

- first-order condition for the poor

$$p^S u'\left(c_0^P\right) = \beta u'\left(c_1^P\right) \quad \underbrace{\lambda\left(\hat{\omega}\right)}_{\text{threshold piece}}$$

Endowment Sales with Private Information

- non-arbitrage:

$$p^S = \mathbb{E}\left[\lambda\left(\omega\right)|\omega < \hat{\omega}\right] p^B$$

- first-order condition of rich:

$$p^B = \beta \frac{u'\left(c_1^R\right)}{u'\left(c_0^R\right)}$$

- first-order condition for the poor

$$p^B = \beta \frac{u'\left(c_1^P\right)}{u'\left(c_0^P\right)} \frac{\lambda\left(\hat{\omega}\right)}{\mathbb{E}\left[\lambda\left(\omega\right)|\omega < \hat{\omega}\right]}$$

Solving it!

- Two unknowns $\left\{p^B, \hat{\omega}\right\}$:

$$\frac{u'\left(c_1^R\right)}{u'\left(c_0^R\right)} = \frac{u'\left(c_1^P\right)}{u'\left(c_0^P\right)} \frac{\lambda\left(\hat{\omega}\right)}{\mathbb{E}\left[\lambda\left(\omega\right)|\omega < \hat{\omega}\right]}$$

$$p^B = \beta \frac{u'\left(c_1^R\right)}{u'\left(c_0^R\right)}.$$

- To solve, plug $\left\{c_0^p, c_1^p, c_0^r, c_1^r\right\}$ into objective:

$$c_0^P = y_0^P + p^B \int_0^{\hat{\omega}} \lambda\left(\omega\right) d\omega$$

$$c_1^P = p^B \int_{\hat{\omega}}^1 \lambda\left(\omega\right) d\omega$$

$$c_0^R = y_0^P - p^B \hat{\omega}$$

$$c_1^R = y_1^R + p^B \int_0^{\hat{\omega}} \lambda\left(\omega\right) d\omega$$

Back to Marina and Pierre

- Gov doesn't satiate market with safe assets
  - would obtain competitive equilibrium
  - doesn't like this, Pareto weights are off
- Instead, wants to exploit inefficient allocation
  - if allocation closer to his solution
- Can we see how it does so in this model?

Back to Marina and Pierre's Model

- Condition for fluctuation in marginal utility:

$$\frac{u'\left(c_1^R\right)}{u'\left(c_0^R\right)} = \frac{u'\left(c_1^P\right)}{u'\left(c_0^P\right)} \frac{\lambda\left(\hat{\omega}\right)}{\mathbb{E}\left[\lambda\left(\omega\right)|\omega < \hat{\omega}\right]}$$

$$p^B = \beta \frac{u'\left(c_1^R\right)}{u'\left(c_0^R\right)}.$$

- To solve, plug $\left\{c_0^p, c_1^p, c_0^r, c_1^r\right\}$ into objective:

$$c_0^P = y_0^P + p^B B + p^B \int_0^{\hat{\omega}} \lambda\left(\omega\right) d\omega$$

$$c_1^P = -B + p^B \int_{\hat{\omega}}^{1} \lambda\left(\omega\right) d\omega$$

$$c_0^R = y_1^R - p^B B + p^B \hat{\omega}$$

$$c_1^R = y_1^P + B + p^B \int_0^{\hat{\omega}} \lambda\left(\omega\right) d\omega$$

Back to Marina and Pierre's Model

- Condition for fluctuation in marginal utility:

$$\Downarrow \frac{u'\left(\Uparrow c_1^R\right)}{u'\left(\Downarrow c_0^R\right)} = \Uparrow \frac{u'\left(\Downarrow c_1^P\right)}{u'\left(\Uparrow c_0^P\right)} \frac{\lambda\left(\hat{\omega}\right)}{\mathbb{E}\left[\lambda\left(\omega\right)|\omega < \hat{\omega}\right]} \Downarrow$$

$$\Downarrow p^B = \Downarrow \beta \frac{u'\left(c_1^R\right)}{u'\left(c_0^R\right)}.$$

- To solve, plug $\left\{c_0^p, c_1^p, c_0^r, c_1^r\right\}$ into objective:

$$\Uparrow c_0^P = y_0^P + p^B \Uparrow B + p^B \int_0^{\hat{\omega}} \lambda\left(\omega\right) d\omega$$

$$\Downarrow c_1^P = - \Uparrow B + p^B \int_{\hat{\omega}}^1 \lambda\left(\omega\right) d\omega$$

$$\Downarrow c_0^R = y_0^R - \left(p^B \Uparrow B + p^B \hat{\omega}\right)$$

$$\Uparrow c_1^R = y_1^P + \Uparrow B + p^B \int_0^{\hat{\omega}} \lambda\left(\omega\right) d\omega$$

Summary of Collateral Example

- In summary:

$$\Downarrow p^B \Rightarrow \text{lower rate}$$

- Partial crowding out:

$$\Downarrow p^B \hat{\omega} \Rightarrow \text{crowd out private credit}$$

- But Goverment doesn't go all the way
  - doesn't want to lower price p^B
  - can have better terms-of-trade for poor (higher p^B)
  - needs inefficiency

Summary

- Paper has a nice insight!
  - Goverment can allow market inefficiencies
  - could have ruled the out,
  - choses to trade off inefficiency for egalitarian
- Inefficiency in example
  - not wastes as in paper
  - but inefficient consumption fluctuation
- Quantitative Model?
  - I think a policy maker like Charlie is happy enough with insight!