Endogenous Technological Growth

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"Every act of creation is first an act of destruction"
Pablo Picasso

1 Introduction

So far we studied how the neoclassical model was successful in delivering all 6 Kaldor facts. All the burden of economic growth was loaded on technology. Technology is an exogenous factor of the model. Essentially, it is a parameter that varies independent of the mechanics of the model. At first, it may seem disappointing to have a theory of growth that is independent by the actions of agents. However, Solow's analysis is extremely valuable because it says that growth must come from a factor that is remunerated, there is no price for it. This points us towards the direction of externalities as a crucial element of growth.

Next, we study two models where these externalities are present. First, we study a class of models labeled Schumpeterian models. Schumpeter had a theory of business cycles based on the idea that innovation by some firms could cause the demise of their competitors and eventually cause economic fluctuations. The Schumpeterian idea is that the creative discovery of a process by one firm destroys the business opportunities for others. However, a more modern terminology is the use of the term "quality ladders". The first model I present is an oversimplistic version of a model by Aghion and Howitt.

The second model is the model of Romer. Romer's model is about varieties. He developed the mathematical details of an idea that was lurking at the time of his writing: the idea that technology involves multiple steps of production and that as we add those steps, we can increase output.

2 The AK model

We abstract away from labor input and have that production is now only a function of the capital stock, and features linear returns:

$$Y_t = A \cdot K_t. \tag{1}$$

The rate of return is:

$$R = A + 1 - \delta. \tag{2}$$

Intuitively, it refers to the amount of new capital stock each unit of capital can generate in a given period: A units of output it produces which can be transfromed costlessly into capital stock and $1 - \delta$ units of the remaining capital net of depreciation.

The investment formula is exactly as before:

$$K_{t+1} = I_t + (1 - \delta) K_{t.} \tag{3}$$

We also obtain a formula the dynamics of consumption. This comes from a fully fledged optimization problem. You would know how to solve this in a masters course.

$$C_{t+1} = \beta R C_t. \tag{4}$$

Notice that consumption grows over time if $\beta R > 1$, i.e. if capital is more productive and people are more patient.

Guess and verify the solution:

$$C_t = (1 - s) Y_t, \tag{5}$$

where s denotes the saving rate as usual.

Now substitute (1), (2) and (5) into (4) to obtain:

$$(1-s)\, AK_{t+1} = \beta\,(A+1-\delta)\,(1-s)\, AK_t. \tag{6}$$

Cancel out $(1-s)$ and A, and now replace (3). We obtain:

$$I_t + (1-\delta)\, K_t = K_{t+1} = \beta\,(A+1-\delta)\, K_t.$$

Once again we replace the investment formula:

$$sAK_t + (1-\delta)\, K_t = \beta\,(A+1-\delta)\, K_t.$$

We can now cancel capital from both sides and obtain:

$$sA + (1-\delta) = \beta\,(A+1-\delta)\,,$$

which yields:

$$s = \left[\beta - \frac{(1-\beta)\,(1-\delta)}{A}\right]. \tag{7}$$

Now we look at whether we can have a growing economy with this model. The condition for growth is:

$$K_{t+1} > K_t,$$

then, from (6) this implies:

$$sA + (1-\delta) > 1.$$

Thus, substituting the formula for the saving rate (7), we obtain:

$$\beta A - (1-\beta)\,(1-\delta) + (1-\delta) > 1.$$

Thus,

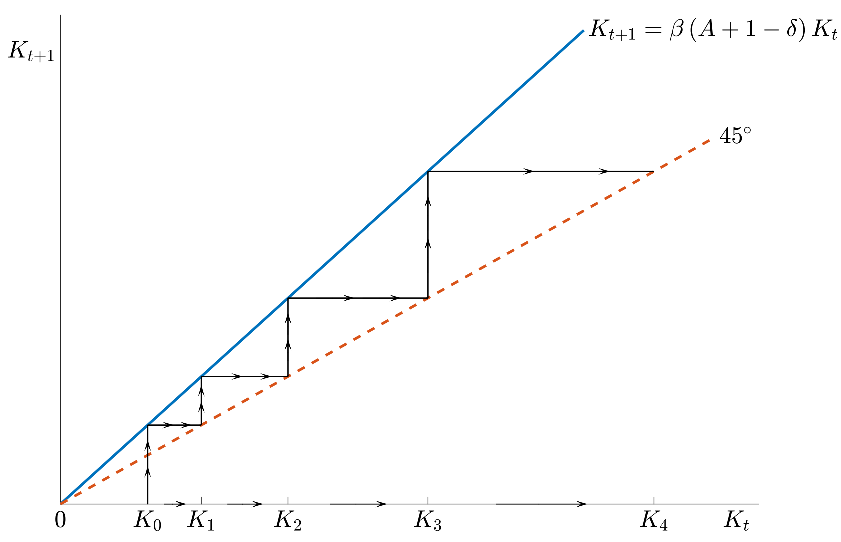
$$\beta\,(A+(1-\delta)) > 1,$$

or simply put:

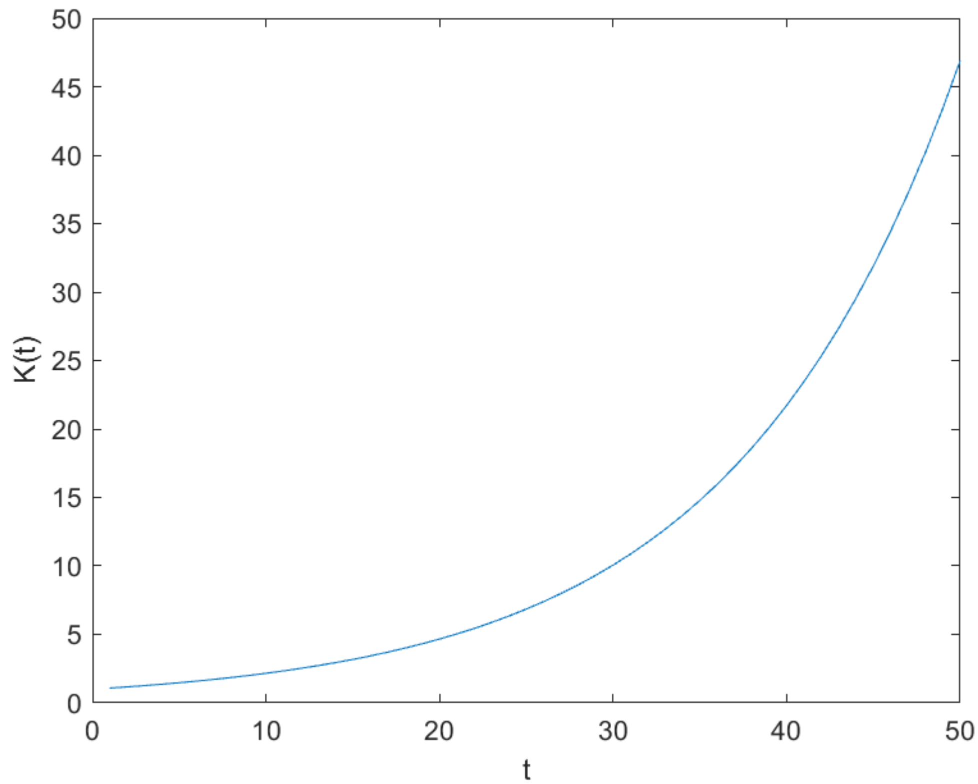
$$\beta R > 1. \tag{8}$$

Therefore, with certain parameter choice, we can model a growing economy with this structure even though the TFP \$A\$ is fixed. The crutial assumption that leads to endogenous growth in this model is that the marginal productivity of capital is constant, whereas in the Solow models we studied in previous lectures it is decreasing with the amount of capital stock.

The graphs below show the transition dymanics of the path of sustained growth with the linear AK technology when condition (8) holds:



and capital growth over time:



Question #1 (Policy Changes). Consider now a policy that taxes income to force some investment. Explain the consequences of such a policy.

HINT: Modify the model and let the equations now be:

$$C_{t+1} = \beta \left(A \left(1 - \tau \right) + \left(1 - \delta \right) \right) C_t,$$

and

$$K_{t+1} = I_t + \tau A K_t + \left(1 - \delta \right) K_t.$$

Discussion. The key advantage of this model is that it leads to perpetual growth. Like one of the requirements of Kaldor, the growth rate is approximately linear. However, there are no transitional dynamics, as in earlier models. This is something that we see in the data. Also, the model is not explicit about what exactly K_t is. Another of the downsides is the lack of labor share. As we see next, this issue can be corrected.

3 Externalities Model

We now consider a version of Romer's (1986) model. Production is given by:

$$Y_t = A K_t^{\alpha} L_t^{1-\alpha} \bar{K}_t^{1-\alpha}.$$

In equilibrium, we assume that:

$$K_t = \bar{K}_t,$$

so $\bar{K}_t$ is proportional to the capital stock.

Next, we determine the wage, by computing the marginal produce of labor. We find that:

$$w_t = \left(1 - \alpha \right) K_t^{\alpha} L_t^{-\alpha} \bar{K}_t^{1-\alpha} = \left(1 - \alpha \right) \bar{K}_t \cdot L_t^{-\alpha}.$$

If we reconstruct the labor share, we obtain that again, it equals:

$$\frac{w_t L_t}{Y_t} = \left(1 - \alpha \right).$$

Now, we proceed to derive the return on capital (from a private perspective). Consider the problem of a firm:

$$\max AK_t^{\alpha}L_t^{1-\alpha}\bar{K}_t^{1-\alpha} - r_tK_t - w_tL_t.$$

Take derivatives:

$$\alpha AK_t^{\alpha-1}L_t^{1-\alpha}\bar{K}_t^{1-\alpha} = r_t \Rightarrow r_t = \alpha AL_t^{1-\alpha}.$$

Note that the rate depends on the population size. This is in my opinion a highly undesirable feature. Anyhow, for the time being, it's convenient to assume that $L_t = 1$. In that case, we have a gross return of

$$R_t = \alpha A + (1-\delta).$$

This is an important result. There's a distinction between private and social (public) returns to capital investment. If we consider the social returns, then we would have:

$$R_t^{soc} = A + (1-\delta),$$

that is, the complete range of social returns.

Question #3. Explain what would happen with the model if we modify the production function to:

$$Y_t = AK_t^{\alpha}L_t^{1-\alpha}\bar{K}_t^{\rho}.$$

What happens when we have the condition $\rho + \alpha > 1$ and what happens if we reverse the inequality?

4 The Schumpeterian Model

First, we study a version without capital accumulation. Then we discuss the role of introducing capital to the model. We let output be determined by:

$$Y_t = A_tL_t,$$

a linear technology. We can think of A_t as the level of TFP available to some or many firms.

Access to Blue Prints. There are property rights over blue prints. A_t is accessible only if you have access to the corresponding blue print. There are two possible states of the world. In a industry that has a clear leader, only one firm can access to the A_t blue print at period t. We call that firm the *leader* or monopolist firm. All other firms will have access to the blue print for A_t only by $t+1$, but for now they can only access a level of technology $A_{t-1} < A_t$. That is, other firms in the market have access to a technology which is inferior to the one of the leader.

In the competitive industry state, where no firm has a technological lead at t, all firms access the same technology from the previous period A_{t-1}.

If the world is in a state with a leader, there's a going to be a transition to an industry without a leader. In particular, for our initial example, the transition will happen after one period.

The Creation and Access to Blue Prints. The important feature of this model is that, as opposed to the neoclassical framework where technology was presented as exogenous, now technology requires resources and grows depending on the resources that society (and in particular innovators) destines to the development of blueprints. When a new blueprint is created, the new technology generates a new TFP

$$A_{t+1} = (1+\gamma)A_t.$$

Here,γ is a parameter that models the growth of technology. The parameter γ also refers to a "level" or technological ladder. The idea is that technologies must be developed in a specific order from less to more sophisticated (e.g. you can't build a mobile phone if you don't have a portable technology for sending wireless signals). So γ represents the jump.

Developing a new blue print requires an investment in resources but it may not be successful. In particular, the firm can decide to employ n people in research and development to have access to the more advanced technology with probability

$$\pi(n).$$

We assume a functional form for $\pi(n)$. The probability of success in R&D is given by:

$$\pi(n) = \nu\frac{n^{1-\sigma}}{1-\sigma}.$$

This probability function models the fact that there are decreasing returns to scale in investing in R&D. ν is just a parameter chosen to make sure that the function above is indeed a probability between 0 and 1.

The n workers that the firm uses in R&D come from the general labor force, so they will be paid a wage w_t.

The Leader's Profit. The profits of any producing firm are:

$$\Pi_t = A_t L_t - w_t L_t.$$

If the leader produces, he gets to access technology A_t at time t, while his competitors have access to A_{t-1}. Thus, the leader can "price-out" his followers by setting:

$$w_t = A_{t-1}.$$

Any wage higher than A_{t-1} will make the less technologically advanced firms drop out of the market because they would make negative profits. But in an equilibrium where competitive markets can be restored, it must be the case that no firm drops out, which leads to the previous result. Now that we know the wage, less technologically advanced firms make zero profits, while the profits for the leader are:

$$\begin{aligned}\Pi_t^j &= A_t L_t - A_{t-1} L_t \\ &= (\gamma A_{t-1}) L_t,\end{aligned}$$

using the fact that $A_t = (1+\gamma) A_{t-1}$.

Thus, the value of the development of a patent can be written in the following way: in period $t-1$ the firm pays a wage w_{t-1} to the n_{t-1} workers employed in R&D and in period t they will get the monopoly profits with probability $\pi(n_{t-1})$. In mathematical terms, the firm will choose n_{t-1} to maximize

$$\begin{aligned}& \beta\pi(n_{t-1})\gamma A_{t-1} L_t - w_{t-1} n_{t-1} \\ = \; & \beta\pi(n_{t-1})\gamma A_{t-1} L_t - A_{t-1} n_{t-1},\end{aligned}$$

where the term β is a discount factor to take period t profits in period $t-1$. In $t-1$ all firms have access to the same technology, which leads to $w_{t-1} = A_{t-1}$. Taking the first-order condition yields:

$$\beta\pi'(n_{t-1})\gamma A_{t-1} L_t = A_{t-1}.$$

Then, n_{t-1} solves:

$$\beta\pi'(n_{t-1})\gamma L_t \quad = \quad 1.$$

Assume that the amount of workers is constant over time, so $L_t = \bar{L}$. Thus:

$$\beta\nu n_{t-1}^{-\sigma}\gamma\bar{L} = 1,$$

which leads to

$$n_{t-1} = \left(\beta\nu\gamma\bar{L}\right)^{1/\sigma}.$$

Replace this value in the probability $\pi(n)$ to obtain the probability of a technological jump

$$\nu\frac{\left(\beta\nu\gamma\bar{L}\right)^{(-\sigma+1)/\sigma}}{(1-\sigma)}.$$

Social Value of Research. What is the value of patent from a social perspective? The cost of investing in R&D is the same, but the benefits are much larger. In fact, society will benefit from innovation for all future periods in the form of higher wages for everyone. To be more specific, let's consider the value of a single step-forward in technology. That is, we start with technology A_{t-1}, we may get to level A_t and then every other innovation in the future fails. Call $\bar{\pi}$ the probability of a new successful innovation in the future beyond A_t. Innovation fails with probability $1-\bar{\pi}$, thus s periods from today innovation always fails with probability $(1-\bar{\pi})^s$. The per period benefit from a social perspective is the same as the profit of the monopolistic firm. When innovation is successful, the benefit is captured by the monopolistic firm as profits. Once the patent becomes public, it will be captured by the increase in wages for all consumers.

Putting all together, the social value of innovation is

$$\text{Social Value}= -A_{t-1}n + \pi(n)\,\beta \sum_{s=0}^{\infty} \left(\beta(1-\bar{\pi})\right)^{s} \gamma A_{t-1}\bar{L}.$$

Note that:

$$\sum_{s=0}^{\infty} \left(\beta(1-\bar{\pi})\right)^{s} = \frac{1}{1-\beta\left(1-\bar{\pi}\right)}.$$

Thus, the objective function of a social planner (i.e. government) is:

$$-A_{t-1}n + \pi(n)\frac{\beta\gamma A_{t-1}\bar{L}}{1-\beta\left(1-\bar{\pi}\right)}.$$

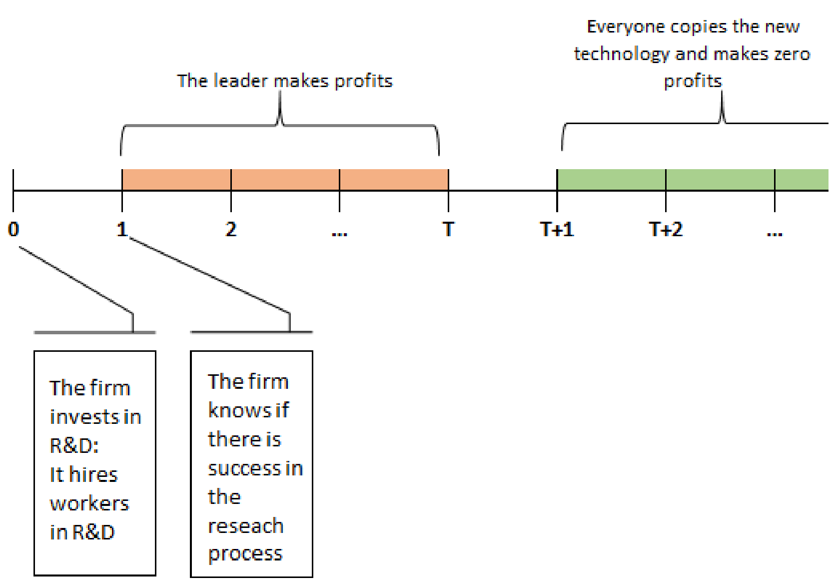
You will recognize that this is similar to the problem of the firm discussed previously. Wherever β appeared, there now is the additional term $\frac{\beta}{1-\beta(1-\bar{\pi})}$. The optimal effort in R&D by the social planner is given by:

$$n_{t-1} = \left(\nu\gamma\bar{L}\frac{\beta}{1-\beta\left(1-\bar{\pi}\right)}\right)^{1/\sigma}$$

which is a much higher number than the private values since $\frac{\beta}{1-\beta(1-\bar{\pi})} > \beta$. This is a reason why the government may want to subsidize programs like the NSF or NASA.

Question #4 (Patent-Life Time). Now assume that agents can keep the patent for T periods. We are interested in the case where $T > 1$. That is, the leader makes profits during the period of time between 1 and T. After that, the followers copy the innovation. Of couse, time here is discrete. Figure 1 summarizes the model.

Figure 1: Timeline of the patent



1. What is the optimal amount of workers n a firm would employ in R&D ? Hint: What are the profits that the firm would make if there is innovation? Remember that the firm discounts future with a factor β. Using this information, you can compute the expected profits from the perspective of the firm. Note that the firm has to pay the wages for the workers in R&D in the initial period (this is independent of the realization of innovation)

2. Consider a planner that takes the choice of the firm as given and only cares about workers' wages from one innovation. This means that this planner uses the choice of n from the previous part, does not take into account the cost of innovation nor the possibility of further innovation, and only considers the wage gains when the patent expires (i.e. from period $T + 1$ onward). What is the social value of one innovation for this planner? Hint: Do not try to copy the formula from the lecture notes. Here you just need to consider the value of wages after the technology becomes public.