Spring 2016 Econ 164 Final Exam

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The exam has 6 questions for 100 points in total. You have 3 hours to finish the exam.

NOTE THAT POINTS VARY BETWEEN QUESTIONS!

DO NOT OPEN THE EXAM BOOK UNTIL THE EXAM BEGINS!

Name:

UID:

Grade:

1. Consider the Solow model with population growth but no technology progress. Suppose at time 0 the population growth rate is suddenly decreased due to a birth control policy. Answer the following questions. (12 points)

a. What is the long run effect on capital per capita of the decrease in population growth rate? Show graphically how you reach the conclusion. (4 points)
A decrease in the population growth rate will increase the capital per capita in steady state, since

$$k_{ss} = \left(\frac{sA}{\delta + n}\right)^{1/1-\alpha}$$

b. What is short run effect on capital per capita? Draw the dynamic path of capital per capita after the reduction in population growth rate. (4 points)
There is no jump in the level of capital. Since we have to go to a higher level of capital per capita, capital accumulation will accelerate at first and then slow down all the way to the new growth rate of population. Therefore in the long run, capital will grow at a smaller rate than before the policy.

c. Draw the dynamic path of the log of aggregate capital for different population growth rates. Show that higher population growth leads to higher aggregate capital in the long run. (4 points)
Intuitively, the higher the population growth rate, the higher will be the growth rate of capital. Therefore an economy that starts from low levels of capital but high population growth will eventually take over an economy that starts with high capital but small labor growth.

2. The law of motion for aggregate capital in the Solow model is given by

$$K_{t+1} = sA_t K_t^{\alpha} L_t^{1-\alpha} + (1-\delta)K_t.$$

Population grows according to $L_{t+1} = (1+n)L_t$ and technology grows according to $A_{t+1} = (1+g)A_t$. Answer the following questions. (12 points)

a. Rewrite the production function in terms of capital, labor, and a labor-augmenting technology factor $\tilde{A}_t$. Find the relationship between $\tilde{A}_t$ and A_t, and the growth rate of $\tilde{A}_t$. (4 points)

The production is

$$Y = K_t^{\alpha} \left(\tilde{A}_t L_t\right)^{1-\alpha}$$

Therefore

$$A_t = \tilde{A}_t^{1-\alpha}$$

and

$$1 + g = (1+\tilde{g})^{1-\alpha}$$

b. Find the law of motion for capital per effective labor $k_t = K_t/(\tilde{A}_t L_t)$. Solve for the steady state of k_t. (4 points)

The law of motion is

$$k_{t+1} = \frac{sk_t^{\alpha}}{1+n+\tilde{g}+n\tilde{g}} + (1-\delta)\frac{k_t}{1+n+\tilde{g}+n\tilde{g}}$$

So the steady state is

$$k_{ss} = \left(\frac{s}{\delta+n+\tilde{g}+n\tilde{g}}\right)^{1/1-\alpha}$$

c. Find an expression for steady state per capita consumption. What is the growth rate of it? (4 points)

Consumption per capita will grow at the growth rate of technology $\tilde{g}$. In fact

$$c = (1-s)\frac{Y}{L} = (1-s)\tilde{A}_t\left(\frac{s}{\delta + n + \tilde{g} + n\tilde{g}}\right)^{\alpha/1-\alpha}$$

3. Government can be a source of low TFP in developing countries. Consider the following problem. There are two sectors in an economy, $s = 1, 2$. They both produce output according to the following technologies:

$$y_1 = A_1^{1-\alpha} l_1^{\alpha},$$

and

$$y_2 = A_2^{1-\alpha} l_2^{\alpha}.$$

The output of both sectors are equally valuable as consumption. Thus, total output is the sum of output of the two sectors:

$$Y = y_1 + y_2.$$

The resource constraint of this economy is given by $l_1 + l_2 = L$, where L is the fixed population of this economy. (20 points)

a. Find out the efficient allocation of this economy? What is the efficient output? (5 points)

Equalize the MPL to get

$$\alpha A_1^{1-\alpha} l_1^{\alpha-1} = \alpha A_2^{1-\alpha} (L - l_1)^{\alpha-1}$$

Or

$$l_1 = \frac{A_1}{A_1 + A_2} L$$

And

$$l_2 = \frac{A_2}{A_1 + A_2} L$$

Efficient output is

$$Y = (A_1 + A_2)^{1-\alpha} L^{\alpha}$$

b. Now suppose the government taxes sector 1 and subsides sector 2 for the output they produce. The tax rate for each unit of sector 1's output is $\tau > 0$ and the subsidy rate for each unit of sector 2's output is $\sigma > 0$. Write down the profit maximi-

zation problem for the two firms. (5 points)
Sector 1 problem is

$$\max\left(1-\tau\right)A_1^{1-\alpha}l_1^{\alpha}-wl_1$$

While sector 2 problem is

$$\max\left(1+\sigma\right)A_2^{1-\alpha}l_2^{\alpha}-wl_2$$

c. Solve for the labor allocation under competitive equilibrium given the tax and subsidy. What is the total output of this economy. (5 points)
Equate the marginal products to get

$$\left(1-\tau\right)\alpha A_1^{1-\alpha}l_1^{\alpha-1}=\left(1+\sigma\right)\alpha A_2^{1-\alpha}\left(L-l_1\right)^{\alpha-1}$$

So

$$\left(1+\sigma\right)^{1/1-\alpha}A_2l_1=\left(1-\tau\right)^{1/1-\alpha}A_1\left(L-l_1\right)\Rightarrow l_1=\frac{\left(1-\tau\right)^{1/1-\alpha}A_1}{\left(1-\tau\right)^{1/1-\alpha}A_1+\left(1+\sigma\right)^{1/1-\alpha}A_2}L$$

And

$$l_2=\frac{\left(1+\sigma\right)^{1/1-\alpha}A_2}{\left(1-\tau\right)^{1/1-\alpha}A_1+\left(1+\sigma\right)^{1/1-\alpha}A_2}L$$

So

$$Y=\frac{\left(1-\tau\right)^{\alpha/1-\alpha}A_1+\left(1+\sigma\right)^{\alpha/1-\alpha}A_2}{\left(\left(1-\tau\right)^{1/1-\alpha}A_1+\left(1+\sigma\right)^{1/1-\alpha}A_2\right)^{\alpha}}L^{\alpha}$$

d. The government has to balance its budget such that the tax income from the sector 1 exactly covers the subsidy to sector 2. From that, find a relationship between σ and τ. (5 points)
It must be that

$$\tau A_1^{1-\alpha}l_1^{\alpha}=\sigma A_2^{1-\alpha}l_2^{\alpha}$$

So

$$\tau\left(1-\tau\right)^{\alpha/1-\alpha}A_1=\sigma\left(1+\sigma\right)^{\alpha/1-\alpha}A_2$$

4. An economy has a fixed capital stock K, which is distributed between two sectors of the economy, $s = 1, 2$. Let's say the people in sector 1 own 2/5 of the capital stock, and people in sector 2 own the rest 3/5. Denote the initial capital allocation as K_i, such that $K_1 = 2/5K$ and $K_2 = 3/5K$. The two sectors produce output according to the following technologies:

$$y_1 = A_1 k_1$$

and

$$y_2 = A_2 k_2,$$

where k_i is the actual capital used by sector i in production, and we have $A_2 > A_1$. Total output produced is the sum of output of the two sectors.

$$Y_t = y_1 + y_2.$$

The economy's feasibility constraint is given by:

$$K = k_1 + k_2, \text{ and } k_1, k_2 \geq 0$$

Answer the following questions. (20 points)

a. What is the efficient allocation of capital? What is the total output when the allocation is efficient? (4 points)

All capital should go to sector 2, so the efficient output

$$Y = A_2 K$$

b. Suppose there is borrowing constraint to each sector, such that

$$b_i \leq \lambda K_i,$$

Show that when $\lambda < \bar{\lambda}$, the competitive economy will not achieve the efficient allocation. Find out $\bar{\lambda}$ given the initial allocation. (10 points)

$\bar{\lambda}$ satisfies

$$\frac{2}{5}K = \bar{\lambda}\frac{3}{5}K$$

Thus

$$\bar{\lambda} = \frac{2}{3}$$

c. Find out the equilibrium interest rate when $\lambda < \bar{\lambda}$. Show that total output is reduced by the presence of the borrowing constraint under this condition. (6 points)
The equilibrium rate is

$$r = A_1$$

And total output is

$$Y = \left[\frac{2-3\lambda}{5}A_1 + \frac{3+3\lambda}{5}A_2\right]K$$

The total TFP in the economy is a weighted average of $A_1$and A_2, which must be smaller than A_2. Thus output is reduced by the borrowing constraint

5. Consider the model of tragedy of commons. Natural resource f_t evolves according to the following rule:

$$f_{t+1} = A\left(f_t\right)^{\alpha} + (1-\delta) f_t - C_t,$$

where C_t is the aggregate consumption of the resource. There are n identical individuals in this economy. The individuals make consumption decisions with cost function $h(c) = 1/2c^2$. Each individual is to maximize:

$$\max_c c - h\left(c\right),$$

where c is the individual consumption. The aggregate consumption is the sum of all individual consumption. Answer the following questions. (16 points)

a. What is the consumption determined by the individuals? What is the aggregate consumption when all individuals make decision on their own? (5 points)

b. What is the optimal amount of aggregate consumption (Golden rule)? (5 points)

c. Suppose the government decides to implement the Golden rule consumption level by levy a tax on consumption. The individuals now face the following problem:

$$\max_c \quad c - h\left(c\right) - tc,$$

where t is the tax rate of consumption. Find out the tax rate that leads to the Golden rule. Show that the more people in this economy, the higher the tax rate. (6 points)

6. Consider a Malthusian economy. The number of births B_t at time t is given by

$$B_t = b_B y_t^{\beta_B} L_t,$$

where y_t is per capita income and L_t is population. Similarly, the number of deaths D_t is given by

$$D_t = b_D y_t^{-\beta_D} L_t$$

The production function in the Malthusian economy is,

$$Y_t = AT^{\alpha} {L_t}^{1-\alpha},$$

where T is the fixed land supply. Answer the following questions. (20 points)

a. Find out the steady state output per capita in this economy. Is it affected by the level of technology? (5 points)

b. Write down the dynamic equation for population. Find out the steady state for population. (5 points)

c. Suppose there is reduction in b_D due to an advance in medicine, such that the number of deaths at all income levels is reduced. What is the long run effect on output per capita of this reduction? Show that an advance in medicine might not be a blessing to a Malthusian economy. (5 points)

d. What is the short run effect of the reduction in b_D? Draw the dynamic path of output per capita and population. (5 points)