Final Exam
UCLA - 2016
ECON 221C MONETARY ECON III
Saki Bigio

Dear Students,

You have exactly 2.5 hours to finish the two questions of this exam. It's worth 100 points total. The exam seems longer than what it really is. Read questions carefully. It should be a fun problem to work out and hopefully you'll learn as much as I did preparing the exam.

Question 1 is about exchange rates, something we didn't cover in class, you require no knowledge on internatinal macro. I will provide definitions in a way that allows you to solve questions without any knowledge of international macro. Question 2 is about a model we learned in class. It should be easy to solve, since it was part of a problem set to students in another program. You are better trained.

Best wishes and good luck,

Saki

QUESTION 1 - 60 points

Some known puzzles in international macro are the following:

1. *Feldstein-Horioka*: the fact that differences in real returns across countries are persistent and that the current-account takes time to close before those differences vanish.

2. Backus-Smith: that consumption is high when the real exchange rate is high, although consumption is five times less volatile than the real-exchange rate.

3. The terms of trade, is three times less volatile than real exchange rate.

4. High interest rates predict appreciations —i.e., there are violations to the uncovered interest rate parity.

This question uses a variation to one of the models used in class to see how far we can go to analyze those puzzles.

The Environment. Consider a single, small economy, populated by entrepreneurs. They are labeled with some identity $j \in [0, 1]$. Preferences in this problem are given by the following:

$$\sum_{t=0}^{\infty} \log \left(c_t^j\right)$$

where c_t^j is the individual's problem. The entrepreneur in the country accumulates physical capital according to the following technology:

$$k_{t+1}^j = i_t^j + \lambda k_t^j$$

where λ is a gross depreciation and investment.

Now, we have an additional variable, a_t^j, which I label domestic absorption by the entrepreneur. This variable is given by:

$$a_t^j = \left(z_t^j\right)^{1-\upsilon} \left(m_t^j\right)^{\upsilon}.$$

The term z_t^j are domestic consumption and m_t^j are imported goods by entrepreneur j. The parameter υ is a parameter that govern's the preference for foreign consumption.

Absortion is split between domestic consumption and investment:

$$i_t^j + c_t^j = a_t^j.$$

The budget constraint of each individual entrepreneur is:

$$q_t b_t^j + z_t^j + p_t m_t^j = \left(\tilde{k}_t^j\right)^{\alpha} - r_t \hat{k}_t^j + p_t b_t^j$$

where p_t is the price of imported goods, and b_t^j is a foreign bond quoted in units of foreign goods. The price q_t is a constant price. Finally, r_t is the return to each unit of capital in the local economy. We assume that each entrepreneur can either run his own firm or rent capital:

$$\tilde{k}_t^j = k_t^j + \hat{k}_t^j$$

so $\hat{k}_t^j$ is the capital rented from other firms so that $\int_0^1 \hat{k}_t^j dj = 0$ and $\tilde{k}_t^j$ is the capital operated invididually.

Domestic output is produced according to a decreasing returns to scale technology $\left(\tilde{k}_t^j\right)^\alpha$. Total output in the country is:

$$Y_t = \int_0^1 y_t^j dj = \int_0^1 \left(\tilde{k}_t^j\right)^\alpha dj$$

and is split between absorption uses and exports:

$$\int_0^1 z_t^j dj + \int_0^1 x_t^j dj = Y_t.$$

Absorption is used to consume and invest where the split is given by:

$$i_t^j + c_t^j = a_t^j.$$

The only random variable in this economy is the price of the foreign good, p_t. In particular, assume that p_t follows a geometric random walk with to growth rates:

$$\log p_{t+1} = \rho \log p_t + \varepsilon_t^p.$$

Where $\varepsilon_t^p \sim N\left(0, \sigma_p\right)$ stands for a normal distribution with parameters $(0, \sigma_p)$. Also, ρ is an autocorrelation. That is, the log of p_{t+1} is an AR(1) process.

Definitions: The terms of trade in this economy are given by:

$$\tau_t = p_t$$

because export prices are 1 —consumption of the domestic good is the numeraire.

The domestic price level is:

$$\bar{p}_t = \frac{p_t^\upsilon}{(1-\upsilon)^{1-\upsilon}\,\nu^\upsilon}.$$

Finally, the real exchange rate is:

$$RER_t = p_t/\bar{p}_t$$

Question 1 (20pts). Show that the domestic economy admits a representative household with the following budget constraint:

$$\frac{q_t}{\bar{p}_t}b_t + k_{t+1} + c_t = \left(\frac{r_t}{\bar{p}_t} + \lambda\right)k_t + \frac{p_t}{\bar{p}_t}b_t.$$

Question 2 (15pts). Impose the following borrowing constraint. Each entrepreneur faces the following constraint:

$$-qb' \leq \frac{\gamma}{(1-\gamma)}\bar{p}k',$$

for some parameter γ. This constraint states that the entrepreneur cannot borrow beyond a multiple of the domestic price of its capital. Derive the corresponding portfolio separation for this problem. Derive a law of motion for capital and debt taking the solution to that portfolio problem as given ω.

Describe the correlation between the real exchange rate and aggregate consumption. How does it vary with λ?

Question 3 (5pts). What is the correlation between terms of trade and real exchange rates:

$$\bar{p}_t = \Upsilon p_t^{\upsilon}$$

where $\Upsilon = \left(\upsilon^{\upsilon}(1-\upsilon)^{1-\upsilon}\right)^{-1}$. Then, we have that:

$$\bar{p}_t = \Upsilon p_t^{\upsilon}.$$

How does it vary with λ?

Question 4 - Balance of Payments Identity (15pts). Consider the budgent constraint:

$$q_t b_t^j + z_t^j + p_t m_t^j = \left(\tilde{k}_t^j\right)^{\alpha} - r_t \hat{k}_t^j + p_t b_t^j.$$

Aggregate this constraint and express it as:

$$\underbrace{(B - q_{t-1}B)}_{\substack{\text{Factor}\\ \text{Payments}}} + \underbrace{X - M}_{\text{Net Export}} = \underbrace{qB' - q_{t-1}B}_{\substack{\text{Capital}\\ \text{Account}}}.$$

Describe the correlation between the components of the the current account and the real exchange rate. How does this relation vary with λ?

Violations to the UIP. Violations to the UIP condition imply:

$$r_{t+1}K_{t+1} = E\left[\frac{\bar{p}_{t+1}}{p_t}\right]\left(\frac{1}{q_t} - 1\right) + \varepsilon_t.$$

where ε_t is obtained from the model. Obtain an equation that characterizes violations to the UIP. Is ε_t correlated with $\bar{p}_{t+1}$? How does it vary with λ?

Question 2 - Endogenous Liquidity

Consider a problem related to the one studied in class.

Preferences: An entrepreneur features the log preferences and discounts the future at rate β:

$$\sum_{t=0}^{\infty} \beta^t \log(c_t),$$

where c_t is consumption, time t.

Technology: The agent produces y_t consumption units according to: $A(h_t + k_t)$ where $A > 1$ is TFP and h_t an input used at t. Producing h_t costs one unit of output.

Capital: The entrepreneur starts with k_t capital units accumulated from the past. However, capital becomes divisible into a continuum of units $\omega \in [0,1]$. Each differs in the depreciation rate. In particular, each unit is associated with depreciation $\delta(\omega) \equiv \phi_1 + \phi_2 \omega$, where (ϕ_1, ϕ_2) are parameters. Entrepreneurs can sell units individually. A sales decision is given by $I[\omega]$. In turn, the entrepreneur can purchase i^b so that his capital stock evolves via:

$$k_{t+1} = i^b + k_t \int_0^1 (1 - I[\omega])(1 - \delta(\omega))\, d\omega.$$

Cash-in-Advance Restriction: The agent must pay for h_t in advance. Hence, he satisfies:

$$h_t \le m_t$$

where m_t is some form of inside money.

Inside Money: Total liquidity is:

$$m_t = \int_0^1 p(\omega)\, I[\omega] k_t d\omega.$$

Budget Constraint: By the end of the period, the budget constraint is:

$$c_t + i_t^b = A(h_t + k_t) + m_t - h_t.$$

The corresponding Bellman equation is:

$$V(k_t) = \max_{\left\{c_t, i_t^b, h_t, I[\omega], m_t\right\}} \log(c_t) + \beta \log(k_{t+1})$$

subject to:

$$c_t + i_t^b + h_t = A(h_t + k_t) + m_t.$$

$$k_{t+1} = i^b + k_t \int_0^1 (1 - I[\omega]) (1 - \delta(\omega)) \, d\omega.$$

$$m_t = \int_0^1 p(\omega) \, I[\omega] k_t d\omega.$$

$$h_t \leq m_t.$$

Question a (5 points): What does it mean that $p(\omega) = 1 - \delta(\omega)$? How about $p(\omega) = 1$ for all ω?

Question b (5 points): Solve the Bellman equation corresponding to $p(\omega) = 1 - \delta(\omega)$.

Question c (10 points): Solve the Bellman equation when $p(\omega)$ is a constant equal to the expected (gross) depreciation per unit sold.

Question d (20 points): What happens with the economy when ϕ_1 is a constant and we increase ϕ_2. Interpret the solution.