Lecture 12: Misallocation

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May 22, 2019

“When plunder becomes a way of for a group of men living together in society, they create for themselves, in the course of time, a legal system that authorizes it and a moral code that glorifies it.”

Frederic Bastiat

“Some people regard private enterprise as a predatory tiger to be shot. Others look on it as a cow they can milk. Not enough people see it as a healthy horse, pulling a sturdy wagon.”

Winston Churchill

“The problem with socialism is that you eventually run out of other people’s money.”

Margaret Thatcher

1 Misallocation

These notes study two models of misallocation of resources. The first has to do with governments or interest groups intervening on the decisions of people. The second one has to do with the lack of property rights in the economy.

1.1 Misallocation 1 - Government Induced

The first is a model of misallocation of inputs. The idea is very simple. Consider a country that has a fixed population L. There are two sectors of the economy, $s = 1, 2$. They both produce output according to the following Cobb-Douglas technologies:

$$y_{1,t} = k_{1,t}^{\alpha} l_{1,t}^{1-\alpha}$$

and

$$y_{2,t} = k_{2,t}^{\alpha} l_{2,t}^{1-\alpha}.$$

The output of both sectors are equally valuable as consumption. Thus, total output can be considered as a direct sum of the two sectors:

$$Y_t = y_{1,t} + y_{2,t}.$$

The Most Efficient Economy. The most efficient economy is the one that produces the most output with the same volume of resources. For simplicity, let's assume that each sector employs one unit of labor, $l_{1,t} = l_{2,t} = 1$, in order to focus on misallocation of capital. Thus, the problem that the government is solving is:

$$\max_{\{k_{1,t}, k_{2,t}\}} Y_t$$

subject to:

$$\begin{aligned} y_{1,t} &= k_{1,t}^{\alpha} \\ y_{2,t} &= k_{2,t}^{\alpha} \\ k_{1,t} + k_{2,t} &= K_t. \end{aligned}$$

Let's now solve for the optimal allocation of capital. We obtain the following, by simple substitution of constraints into the objective:

$$\max_{\{k_{1,t}\}} k_{1,t}^{\alpha} + (K_t - k_{1,t})^{\alpha}.$$

The solution to this problem is given by:

$$k_{1,t} = k_{2,t} = K/2.$$

Question. Show that this is indeed the case.

And therefore most efficient output in this economy is equal to

$$Y_{eff} = (K_t/2)^{\alpha} + (K_t/2)^{\alpha} = 2^{1-\alpha} K_t^{\alpha}$$

$2^{1-\alpha}$is the TFP of the most efficient economy.

We now show that the optimal allocation obtained in competitive markets is the same - that is, First Fundamental Theorem of economics holds in this economy.

A Competitive Economy. In a competitive economy, we have the following conditions that characterize the optimal choice by the firms:

$$\max_{\{k_{1,t}, l_{1,t}\}} y_{1,t} - w_t l_{1,t} - r_t k_{1,t}.$$

$$\max_{\{k_{2,t}, l_{2,t}\}} y_{2,t} - w_t l_{2,t} - r_t k_{2,t}.$$

Taking First Order Conditions with respect to capital and applying the fact that each sector employs one unit of labor, the demand for capital by each firm is given by:

$$r_t = MPK_{1,t} = \alpha l_{1,t}^{1-\alpha} k_{1,t}^{\alpha-1} \Rightarrow r_t = \alpha k_{1,t}^{\alpha-1} \Rightarrow k_{1,t}(r_t) = \left(\frac{\alpha}{r_t}\right)^{\frac{1}{1-\alpha}}$$

$$r_t = MPK_{2,t} = \alpha l_{2,t}^{1-\alpha} k_{2,t}^{\alpha-1} \Rightarrow r_t = \alpha k_{2,t}^{\alpha-1} \Rightarrow k_{2,t}(r_t) = \left(\frac{\alpha}{r_t}\right)^{\frac{1}{1-\alpha}}$$

Dividing $k_{1,t}(r_t)$ by $k_{2,t}(r_t)$implies

$$\frac{k_{1,t}}{k_{2,t}} = \frac{k_{1,t}(r_t)}{k_{2,t}(r_t)} = 1$$

And thus from market clearing condition for capital (supply of capital K_t has to be equal to total demand $k_{1,t}(r_t) + k_{2,t}(r_t)$) :

$$k_{2,t} = k_{1,t} = \frac{K_t}{2}.$$

so, output (GDP) will also be equal to most efficient output in the competitive economy.

An economy with Misallocation. Now let's consider an economy where labor is misallocated because sector 2 receives a interest rate subsidy equal to $\sigma^{\alpha-1} < 1$ (for some $\sigma > 1$), so that acquiring capital is cheaper for sector 2, and the first sector pays it with a tax equal to $\tau^{\alpha-1} > 1$. In this case, the firm's first-order condition in profit maximization will change.

Let's look first at the firm in sector 2. Sector 2 has the following profit function that it tries to maximize (again, we set labor equal to one):

$$k_{2,t}^{\alpha} - w_t - \sigma^{\alpha-1} r_t k_{2,t}.$$

Let's now consider what happens if the sector wants to maximize profits. The first-order condition in the second sector becomes:

$$\begin{aligned} \sigma^{\alpha-1} r_t &= MPK_{2,t} = \alpha k_{2,t}^{\alpha-1} \rightarrow \\ \left[\frac{\alpha}{r_t}\right]^{\frac{1}{1-\alpha}} &= \frac{k_{2,t}}{\sigma}. \end{aligned}$$

Following the same analytical steps, we obtain that:

$$\tau^{\alpha-1} r_t = MPK_{1,t} = \alpha k_{1,t}^{\alpha-1}.$$

Therefore:

$$\left[\frac{\alpha}{r_t}\right]^{\frac{1}{1-\alpha}} = \frac{k_{1,t}}{\tau}.$$

Thus, we obtain the following relationship:

$$\frac{k_{1,t}}{\tau} = \frac{k_{2,t}}{\sigma}.$$

Observe how the capital decisions are distorted in this economy. We will solve the equilibrium, and compute output without any reference to the relationship between τ and σ. We do that below. For now, we just solve for the capital in each sector, using the equation above and $K_t = k_{1,t} + k_{2,t}$:

$$\begin{aligned} k_{1,t}\frac{\sigma}{\tau} &= K - k_{1,t} \rightarrow \\ k_{1,t} &= \frac{\tau}{\sigma+\tau}K_t, \quad k_{2,t} = \frac{\sigma}{\sigma+\tau}K_t. \end{aligned}$$

Therefore, total output is given by:

$$\begin{aligned} Y_{dist} &= k_{1,t}^{\alpha} + k_{2,t}^{\alpha} \\ &= \left(\frac{\tau}{\sigma+\tau}\right)^{\alpha} K_t^{\alpha} + \left(\frac{\sigma}{\sigma+\tau}\right)^{\alpha} K_t^{\alpha}. \end{aligned}$$

Since $\frac{\tau}{\sigma+\tau}$ and $\frac{\sigma}{\sigma+\tau}$ add up to one, we can reformulate this expression as follows:

$$Y_{dist} = \underbrace{\left[\left(\frac{1}{2}-\mu\right)^{\alpha} + \left(\frac{1}{2}+\mu\right)^{\alpha}\right]}_{\text{TFP}} K_t^{\alpha}$$

where $\mu = \frac{1}{2} + \frac{\sigma}{\sigma+\tau}$ is a parameter that measures the amount of misallocation in the economy. As we saw before, the ideal world is one where $\mu = 0$. This variable describes the extent of misallocation in this economy. The higher the misallocation (the higher μ), the lower is output in the economy.

Notice how TFP is an endogenous variable that depends on the level of missallo-

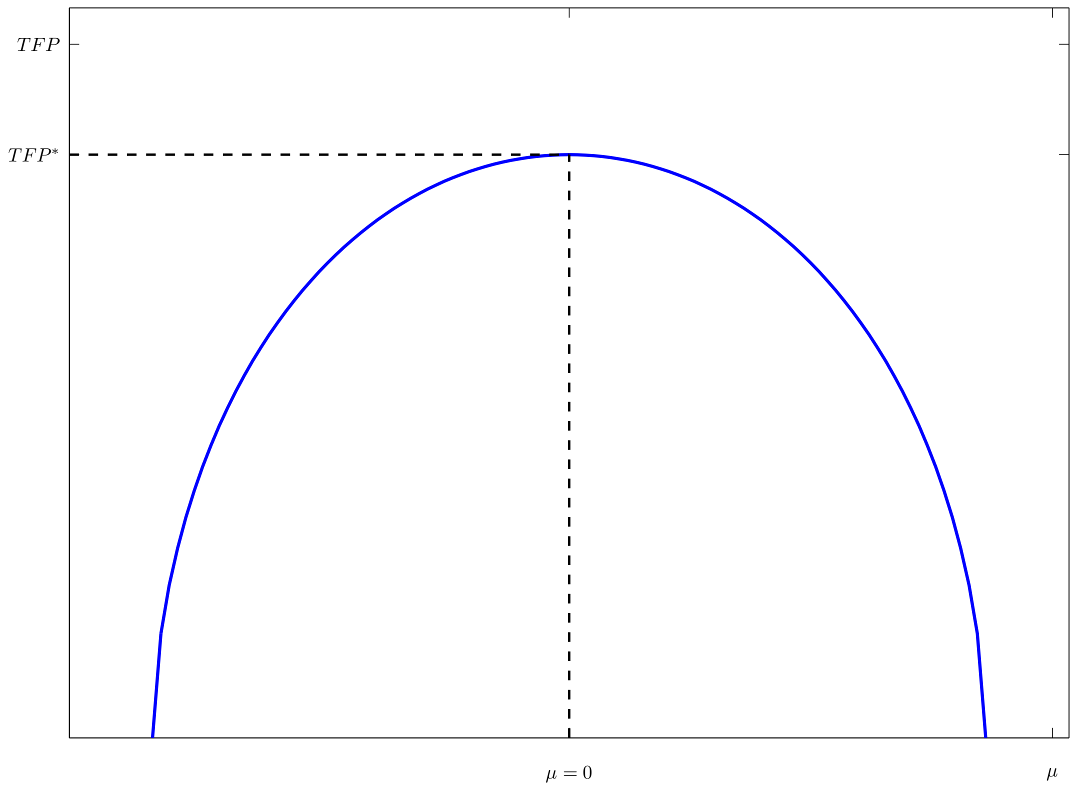


Figure 1: TFP as a function of μ

cation.

$$A(\mu) = \underbrace{\left[\left(\frac{1}{2} - \mu\right)^{\alpha} + \left(\frac{1}{2} + \mu\right)^{\alpha}\right]}_{\text{TFP}}$$

This function reaches a maximum at 0, and then is symmetrically worse and worse. Figure 1 graphically shows the relationship between TFP and the level of distortion μ.

Budget Financing. Until now, we haven't said anything about how to balance the budget. We could argue that there is a deficit and simplify the analysis, or we could make the subsidy be fully funded by the tax in the other sector. In that case, we need to equate:

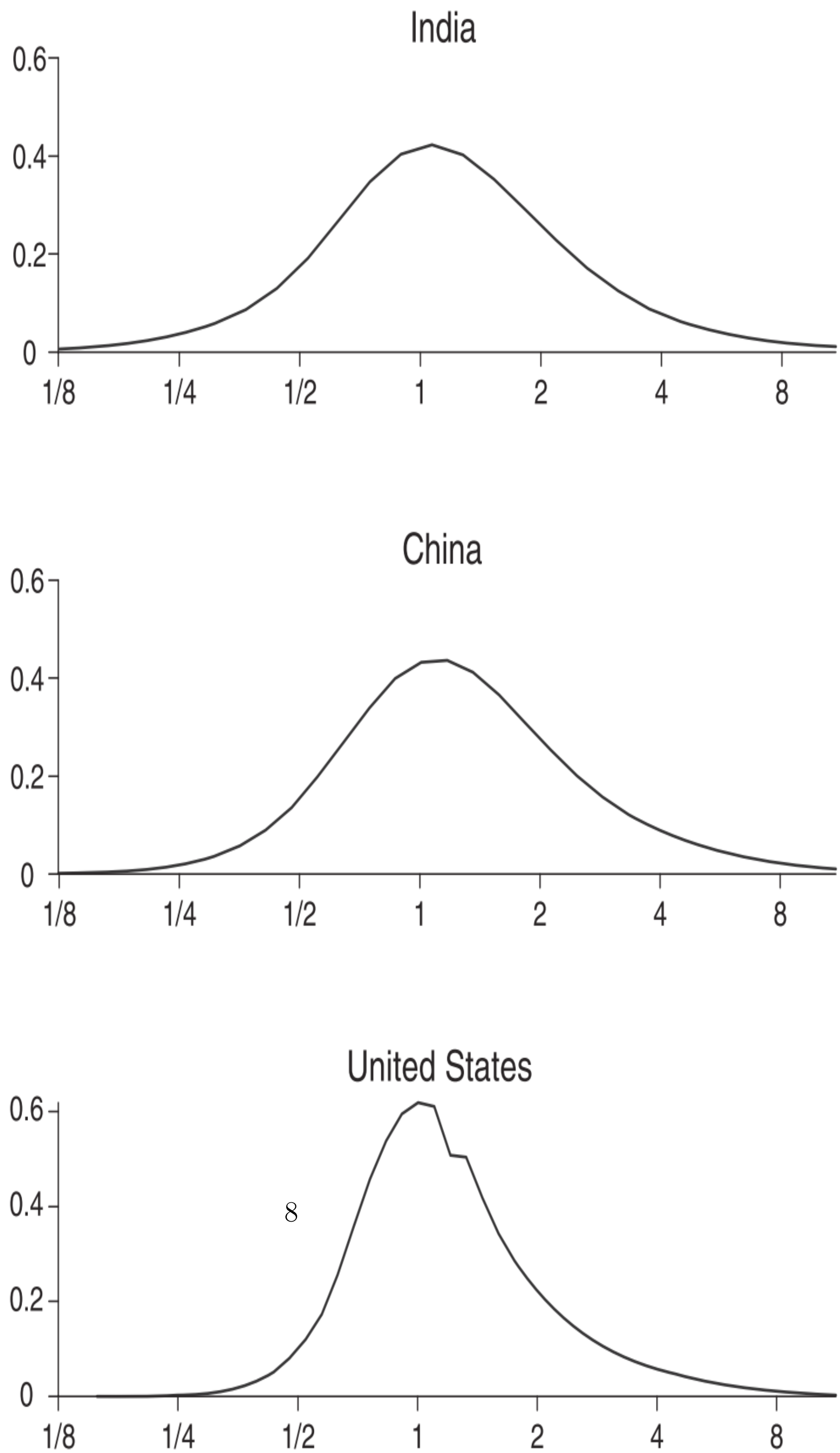
$$\left(1-\sigma^{\alpha-1}\right) r_t k_{1,t} = \left(\tau^{\alpha-1}-1\right) r_t k_{2,t}.$$

Thus, using our solution for volumes of capital in both sectors we obtain:

$$\begin{aligned} \left(1-\sigma^{\alpha-1}\right)\frac{\tau}{\sigma} &= \left(\tau^{\alpha-1}-1\right) \rightarrow \\ \frac{\left(\sigma-\sigma^{\alpha}\right)}{\sigma^2} &= \frac{\left(\tau^{\alpha}-\tau\right)}{\tau^2}. \end{aligned}$$

Hsieh and Klenow evidence. Recent work by Hsieh and Klenow has presented evidence on missallocation across the US, India and China. The idea in that paper is that comparing the distribution of marginal productivities among plants in the US relative to what you see in China and India can give us an idea of how distorted these economies are.

Hsieh and Klenow (2009) estimate that this could account for lower aggregate productivity, almost increasing TFP in China by 30%-50% and in India by 40%-60% (which would also imply comparable or twice as large output gains depending on whether capital at the plant level responds. Figure 2 illustrates where these results come from. It plots the distribution of TFPR for the three countries. We haven't discussed what TFPR is, but in our model, it is just the marginal product of each sectors. Notice that efficient allocation requires marginal product to be equated across sectors, such that the distribution of marginal product gives a measure of distortion in that economy. We can see clearly that the dispersion is larger in China and India than in US. This explains why US is more productive than the other two countries.


India
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China
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1/2
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2
4
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United States
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8

1.2 Misallocation 2 - Lack of Government

The previous analysis shows how the economy can be distorted by a system of subsidies and taxes. It highlights how different interest groups can have incentives to tax other sectors. This section does something different. It shows how the lack of financial markets can be a source of distortion as well. The notes are a simplified version of a model by Benjamin Moll (AER, 2014).

1.2.1 The Efficient Economy

An efficient economy. Consider a country that has a fixed capital stock K. We adopt this assumption because we are thinking of short-run dynamics. There are two sectors of the economy, $s = 1, 2$. The most important difference now is that people in sector 1 own 1/2 of the capital stock. People in sector 2 own the other half. What makes sectors different is their TFP. One sector will be much better at producing than the other.

The production in both sectors is carried out very similarly. Both sectors produce output according to the following technologies:

$$y_{1,t} = A_{1,t}k_{1,t}$$

and

$$y_{2,t} = A_{2,t}k_{2,t}.$$

Here, $A_{2,t} > A_{1,t}$. We have assumed that labor is not used in either sector, but that assumption is easy to be changed. Let's compute the allocation of capital among the two sectors that can produce the highest amount of output.

$$Y_t = y_{1,t} + y_{2,t}$$

and subject to:

$$y_{1,t} = A_{1,t}k_{1,t} \text{ and } y_{2,t} = A_{2,t}k_{2,t}.$$

and subject to the economy's feasibility constraint:

$$K = k_{1,t} + k_{2,t} \text{ and } k_{1,t}, k_{2,t} \geq 0$$

Replacing the units of capital in sector 2, we obtain:

$$\begin{aligned} Y_t &= A_{1,t}k_{1,t} + A_{2,t}k_{2,t} \\ &= A_{1,t}k_{1,t} + A_{2,t}\left(K - k_{1,t}\right) \\ &= A_{2,t}K - (A_{2,t} - A_{1,t})k_{1,t}. \end{aligned}$$

Thus, if we want to maximize output, since $(A_{2,t} - A_{1,t}) > 0$, the best that can happen in this economy, is that all the capital ends operated in the second sector. Thus, $k_{1,t} = 0$.

A competitive economy with perfect property rights. We have obtained the optimal allocation of capital. Now assume that the owner of capital in each sector can produce directly or rent capital. Recall that each of them own 1/2 of the capital stock. Thus, they maximize the following problem:

$$\begin{aligned} \pi_{1,t} &= y_{1,t} + r_t\left(s_{1,t} - b_{1,t}\right) \\ k_{1,t} &= \frac{1}{2}K + b_{1,t} - s_{1,t} \\ y_{1,t} &= A_{1,t}k_{1,t} \\ s_{1,t} &\leq 1/2K. \end{aligned}$$

Each equation in this problem deserves some attention. The first equation is the profit in sector 1. It is the sum of revenues produced in the sector, plus rents received and paid out. In particular, $s_{1,t}$ are units of capital rented to sector 2 by sector 1. Instead, $b_{1,t}$ are the units rented by sector 1. Note that the negative sign in front of r_t stands for amounts paid out. The second equation states how much capital is employed in sector 1. The first entry is $\frac{1}{2}K$, the sector's ownership of the capital stock. The term $b_{1,t}$ stands for the units borrowed which could be added to the sector's stock. Finally, we subtract $s_{1,t}$, because those units are rented out. The units employed in the sector are used in production $y_{1,t}$. The final constraint says that a sector cannot rent out more capital than what it owns (that would mean it is borrowing).

For the second sector, we have the same problem:

$$
\begin{aligned}
\pi_{2,t} &= y_{2,t} + r_t \left(s_{2,t} - b_{2,t}\right) \\
k_{2,t} &= \frac{1}{2}K + b_{2,t} - s_{2,t} \\
y_{2,t} &= A_{2,t} k_{2,t} \\
s_{2,t} &\leq \frac{1}{2}K.
\end{aligned}
$$

An equilibrium in this economy, is a set of allocations: $\{b_{1,t}, s_{1,t}, b_{2,t}, s_{2,t}, k_{1,t}, k_{2,t}, y_{1,t}, y_{2,t}\}$ together with a rental rate for capital r_t , such that:

- Given r_t, $\{b_{i,t}, s_{i,t}\}$ solve the profit maximization problem of each sector.
- The capital market clears:

$$s_{1,t} = b_{2,t} \text{ and } s_{2,t} = b_{1,t}.$$

We now show that the prevalent interest rate in this economy is $r_t = A_{2,t}$, and that the allocation is efficient. For this we transform the problem in each sector. We begin studying the problem in sector 1. If we replace all of the constraints, except the last one, we obtain:

$$
\begin{aligned}
\pi_{1,t} &= \max_{\{s_{1,t}, b_{1,t}\}} A_{1,t}\left(\frac{1}{2}K + b_{1,t} - s_{1,t}\right) + r_t \left(s_{1,t} - b_{1,t}\right) \\
&= \frac{1}{2}A_{1,t}K + \left(A_{1,t} - r_t\right)\left(b_{1,t} - s_{2,t}\right) \\
s_{1,t} &\leq \frac{1}{2}K.
\end{aligned}
$$

By analogy, the objective in sector 2 is:

$$\begin{aligned}\pi_{2,t} &= \max_{\{s_{2,t},b_{2,t}\}} \frac{1}{2}A_{2,t}K + (A_{2,t} - r_t)(b_{2,t} - s_{2,t}) \quad \text{subject to} \\ s_{2,t} &\leq \frac{1}{2}K.\end{aligned}$$

Let's see what happens. The interest rate r_t determines what each sector wants do to. There are a couple of cases:

1. If $r_t < A_{1,t}$, then $(A_{1,t} - r_t) > 0$ but also $(A_{2,t} - r_t) > 0$. This is a problem because it means that both sectors would like to set $b_{i,t}$ as high as possible —borrow as much capital as they can. In turn, they want to sell 0 capital each. This is a problem because if both sectors want to borrow capital, but none want to rent out capital, it cannot be an equilibrium.

2. If $r_t > A_{2,t}$, by the same arguments, neither sector wants to produce. They would both want to rent out capital, but there's nobody to rent capital from.

3. Now assume that $A_{1,t} \leq r_t < A_{2,t}$. In that case, sector 1 wants to lend the most possible: $s_{1,t} = 1/2K$. However, sector 2 wants to rent as much capital as possible. In particular, it would want to rent an infinite amount of capital and make infinite profits. Thus, $b_{2,t} = \infty$ but $s_{1,t} = 1/2K$. Thus, this case cannot be an equilibrium.

4. We are left with only one case: $r_t = A_{2,t}$. In this case, sector 1 still wants to rent out $s_{1,t} = 1/2K$, and sector 2 is ok with renting that amount. To see this, observe that:
$$\pi_{2,t} = \max_{\{s_{2,t},b_{2,t}\}} \frac{1}{2}A_{2,t}K$$
are the profits for sector 2, for any level of $s_{2,t}, b_{2,t}$.

With this, we have shown that the only equilibrium is $s_{2,t} = b_{1,t} = 0$, $b_{2,t} = s_{1,t} = \frac{1}{2}K$. This means that all the capital is operated by the most efficient sector.

1.2.2 The Economy without Financial development

Assume now that each sector faces an additional constraint. The constraint says that the borrowings of each sector are limited by:

$$b_{i,t} \leq \frac{\lambda}{2}K.$$

Here, λ measures how much firms can borrow proportional to their holdings. We assume that $\lambda < 1$. You can think of λ as measuring the amount of collateral in one economy. This can be associated with the amount of property rights in the economy. We will now find that the equilibrium in this economy is:

$$r_t = A_{1,t}.$$

with $b_{2,t} = \frac{\lambda}{2}K$.

We can perform the exact analysis as before, but in step 4, we would find that $b_{2,t} = \frac{\lambda}{2}K$ whereas $s_{1,t} = \frac{1}{2}K$. In step 3, for any value of $A_{1,t} < r_t < A_{2,t}$ we would ran into the same problem. Finally, for a value of $A_{1,t} = r_t$, we would find a difference.

If we substitute this result, we find that output is given by:

$$Y_t = \left[A_{2,t}\left(\frac{1+\lambda}{2}\right) + A_{1,t}\left(\frac{1-\lambda}{2}\right)\right]K.$$

This formula is very similar to the solution we discovered before. The main difference is that it is a weighted average of the two levels of TFP, as opposed to the non-linear formula.

This experiment teaches us the lesson that limiting property rights can limit the economy's potential because it doesn't allow the appropriate placement of resources in the economy. A problem, however, with this theory, is that returns in the second sector can be very high, and thus, over time, that sector can potentially overcome its lack of resources in the short run. That is, if the sector can save.