Lecture 14: Sustainable Growth

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May 24, 2016

1 E-Growth

In this lecture we will discuss a version of a very famous problem in economics, and we will relate it to the course we have been studying so far. The model I refer to is called the fisheries problem, studied many years ago by Levhari and Mirman, about the tragedy of the commons.

1.1 The Great Fish Wars

Suppose there is a natural resource that is renewable —we will adapt the model to describe what happens when the resource is non-renewable. Let's begin by assuming that the resource is indeed renewable. We can think of these resources as being trees, or fish or water at a well that is replenished at a certain rate —unless the well dries up and then there's erosion. Since the problem was originally formulated in terms of fish, we will use the letter f.

The Birth of Fish. Suppose that there is an amount f evolves according to the following rule:

$$f_{t+1} = A\left(f_t\right)^{\alpha} + (1-\delta)f_t - c.$$

Thus, f_t looks pretty similar to the law of motion for capital that we've studied in many models. However, what is different here, is that consumption of fish will depend on something else —it won't be necessarily proportional to Af_t^{α}.

The Extraction of Fish. Now suppose that every period, many individuals can extract fish independently. They are all identical, so they will take the same action. For that reason, we focus on the problem of a single individual that chooses a fishing rate c. It is costly to fish, and it costs h units of consumption. This function satisfies $h' > 0$ and $h'' > 0$. Thus, the individual maximizes:

$$\max_{c} c - h\left(c\right)$$

Note that this problem does not depend on the amount of fish available. In more complicated settings, we could let that be the case, but the essence of the problem is exactly the same. Now we solve the individual decision to fish:

$$0 = 1 - h'(c).$$

Thus, the consumption rate is given by some value of c^*. For example, if we let $h(c) = 1/2c^2$, we have that this problem is solved by $c^* = 1$.

Let's go back to the law of motion for fish. We thus obtain the following rule:

$$f_{t+1} = Af_t^\alpha + (1-\delta)f_t - c^*.$$

Something surprising happens with this model. The dynamics depend on where one starts, and on the value of c^*. It is convenient to adopt the continuous time approach we have worked with in previous lectures. Thus, we obtain:

$$\begin{aligned} f_{t+\Delta} &= \Delta A f_t^\alpha + (1-\delta\Delta)f_t - \Delta c^* \rightarrow \\ f_{t+\Delta} - f_t &= \Delta A f_t^\alpha - \delta\Delta f_t - \Delta c^* \rightarrow \\ (f_{t+\Delta} - f_t)/\Delta &= A f_t^\alpha - \delta f_t - c^* \end{aligned}$$

Case 1. Depletion of the natural resources. Assume that we have:

$$c^* > \max A f_t^\alpha - \delta f_t.$$

Then, we have that for any level of f, the change is negative. The resource is depleted over time.

Case 2. Depletion of the natural resources. Assume that we have:

$$c^* \leq \max A f_t^\alpha - \delta f_t.$$

Then, we have that if f is very small, the change is negative. However, if the resource is found in plentiful amounts, the resource can be reproduced over time.

The Social Optimum. We now study the social optimum. How do we obtain it? The problem requires some amount. However, we know that the solution must be some sustainable amount $c^* \leq \max A f_t^\alpha - \delta f_t$. We can apply the golden rule we studied with the Solow model and improve things:

$$c^* = (1-\alpha)A^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

Exhaustible resources. Next, we ask what happes if the resource is exhaustible? In that case we would set $A = 0$. The logic of the model with exhaustible resources is the same. The social optimum, in general, is related to the hoteling rule. It says that the resource must be extracted at the rate such that it's price follows the rate of interest rates. Clearly, c^* does not depend on this variable.