Lecture 4: Malthusian Mechanics

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"I do not know that any writer has supposed that on this earth man will ultimately be able to live without food."

Thomas Malthus

"Population, when unchecked, increases in a geometrical ratio. Subsistence increases only in an arithmetical ratio."

Thomas Malthus

"Consider this. It took the earth's population thousand of years-from the early dawn of man all the way to the early 1800s-to reach one billion people. Then astoundingly, it took only about a hundred years to double the population to two billion in the 1920s. After that, it took a mere fifty years for the population to double again to four billion in the 1970s. As you can imagine, we're well on track to reach eight billion very soon. Just today, the human race added another quarter-million people to planet Earth. A quarter million. And this happens ever day-rain or shine. Currently every year we're adding the equivalent of the entire country of Germany."

Dan Brown, Inferno

1 Savings Traps

So far we treated the savings rate in the neoclassical model as a constant function of income or wealth. In contexts, this assumption doesn't seem like the best idea because people may find it hard to save, either because for low levels of output, there's a minimum consumption level for subsistence or because there aren't many good opportunities to invest.

We now modify the value of s to be a function of output. We return to the assumption of no TFP growth and no labor growth and we only focus on the dynamics of capital accumulation. We assume that it satisfies:

$$s(y_t) = \begin{cases} s_L \frac{y_t}{y_o} \text{ if } \frac{y_t}{y_o} < 1 \\ \\ s_H \text{ if } \frac{y_t}{y_o} \geq 1 \end{cases} .$$

Here, y_t is output per worker. The idea of this equation is that if $y_t > y_o$, we have that:

$$s\left(y_t\right) = s_H$$

so we have the standard investment rule of Solow model, that is, $s_H y_t$. Instead, if $y_t < y_o$, we have the following lower saving rate savings rate:

$$s_L < s_H$$

meaning that the investment rate is lower than above the cut-off s. Now, total investment is in the limited region:

$$s_L \frac{y_t^2}{y_o}$$

Recall the description of the capital-per-worker accumulation equation,

$$k_t = s\left(y_t\right) A k_t^{\alpha} + \left(1-\delta\right) k_t.$$

Therefore, the capital accumulation equation whenever $y_t < y_o$ yields:

$$k_t = s_L \frac{A^2 k_t^{\alpha 2}}{y_o} + \left(1-\delta\right) k_t \text{ if } A_t k_t^{\alpha} = y_t \leq y_o$$

and

$$k_t = s_L \frac{A_t k_t^{\alpha}}{y_o} + \left(1-\delta\right) k_t \text{ if } A_t k_t^{\alpha} \geq y_o$$

Finding the Good Steady State. Let's see if we can find a good steady statem, one for which $Ak_{ss}^{\alpha} \geq y_o$. The solution for the steady-state capital stock is:

$$k_{ss} = \left[\frac{s_H A}{\delta}\right]^{\frac{1}{1-\alpha}}.$$

This is the same formula that we saw earlier except that it is only valid if $Ak_{ss}^{\alpha} \geq y_o$. For that steady-state to exists, we must verify that:

$$y_o \leq A^{\frac{1}{1-\alpha}} \left[\frac{s_H}{\delta}\right]^{\frac{\alpha}{1-\alpha}}.$$

Finding the Bad Steady State. There exists a "bad" steady state if we are at a steady state and $Ak_{bs}^{\alpha} = y_{bs} \leq y_o$. That steady-state exists if we can find the following solution:

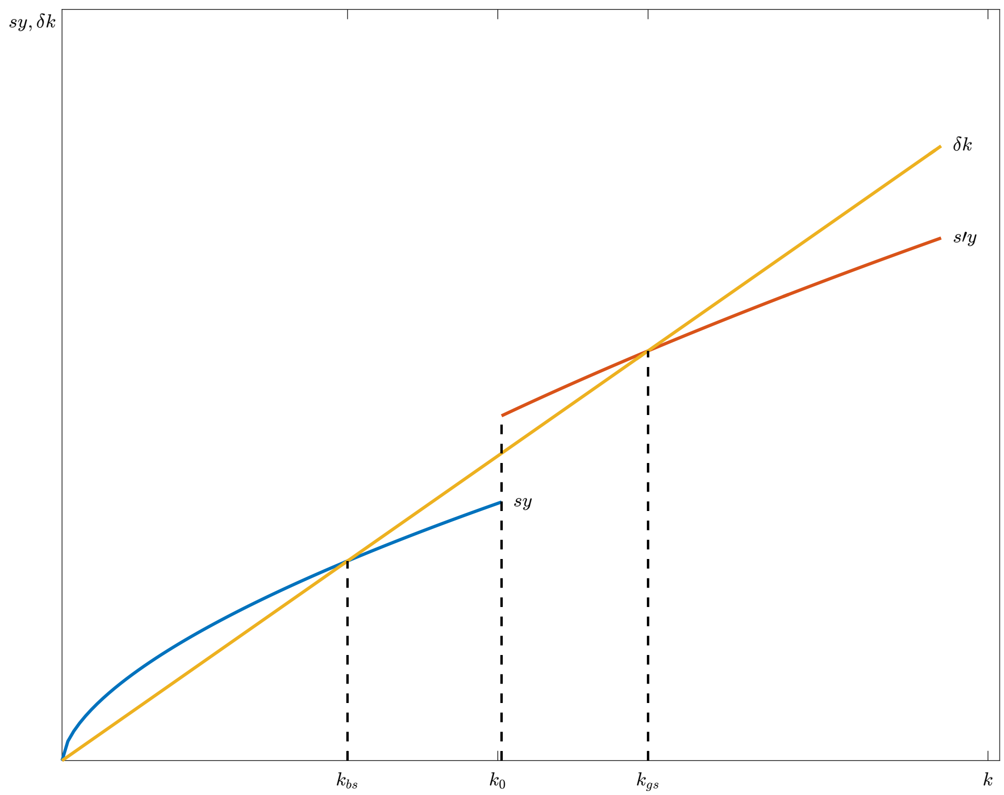

Figure 1: Possibility of Poverty Trap

$$
\begin{aligned}
k_{bs} &= s_H \frac{\left(A k_{bs}^{\alpha}\right)^2}{y_o} + (1-\delta)\, k_{bs} \rightarrow \\
k_{bs} &= \frac{s_L \frac{A^2 k_{bs}^{2\alpha}}{y_o}}{\delta} \rightarrow \\
k_{bs} &= \left[\frac{s_L A^2}{\delta y_o}\right]^{\frac{1}{1-2\alpha}} \\
y_{bs} &= A^{\frac{1}{1-2\alpha}} \left[\frac{s_L}{\delta y_o}\right]^{\frac{\alpha}{1-2\alpha}}.
\end{aligned}
$$

Thus, if $y_{bs} \leq y_o$, this equilibrium is also a steady state.

By using a graphical device we can obtain some surprising conclusions: Figure 1 shows the modified Solow graph. When the economy's capital stock is below k_0, which means it's GDP per capita is below y_0, gross savings of this economy is depicted by the blue line. When the economy's capital stock goes beyond k_0, gross saving is depicted by the green line. Let's look at the fact that there are multiple equilibria. The conclusions of the neoclassical version of Solow's model break down! In particular, we find multiple equilibria. One of the equilibrium implies that there's is low income per capita and the other implies there is high income per capita.

2 The Malthusian Mechanics

Thomas Robert Malthus, an English cleric was the proponent of a theory that predicted stagnated growth. There are many simple formulations of the Malthusian dynamics, so it is good for understanding population from a historical, rather than a modern perspective. As opposed to Solow's model, Malthus's model proposes that population growth rates are endogenous and more importantly, they increase with welfare. Technological advances are therefore translated into more people and effects act as pulling down growth rate per capita because populations are bigger. The model has the following characteristics:

Births. The birth-rate of the population follows:

$$B_t = b_B y_t^{\beta_B} L_t$$

Where B_t is the number of births in the population and y_t, is per capita income. The function: $b_B y_t^{\beta_B}$ expresses the idea that the number of births in population will depend on per capita income. This says that the rates of birth depend on per capita levels, regardless of the total amount of population. A similar equation is found for the number of deaths:

Deaths. Deaths fall with income:

$$D_t = b_D y_t^{-\beta_D} L_t$$

Instead of capital, the model depends on the total endowment of land which is not created or does not depreciate. This is again an argument difficult to defend because land has expanded. The Incan empire for example expanded its arable land to the mountains by the construction of andens, which are huge steps carved in the mountains that make agriculture feasible, and allow the cultivation of diversified products to the differences in latitude. The Dutch expanded their mass of land by building ditches. Land is obtained from the Amazon and it erodes. Regardless of these assumptions, Malthus's model conclusion don't change at all.

Output. The production function assumed by Malthus should have looked like:

$$Y_t = AT^{\alpha} {L_t}^{1-\alpha}$$

so in per-capita terms we obtain:

$$y_t = A(T/L_t)^{\alpha}$$

Demographics. Let's perform a simple accounting rule. Let's count how the population moves over time.

$$\begin{aligned} L_{t+1} &= L_t + B_t - D_t \\ &= \left(1 + b_B y_t^{\beta_B} - b_D y_t^{-\beta_D}\right) L_t. \end{aligned}$$

A Steady State. In a steady-state, it must hold that births and deaths cancel each other —they have to add up to zero.

$$\begin{aligned} B_{ss} &= D_{ss}. \\ b_B \left(y_{ss}\right)^{\beta_B} &= b_D \left(y_{ss}\right)^{-\beta_D} \\ y_{ss} &= \left(b_D / b_B\right)^{1/(\beta_B + \beta_D)} \\ L_{ss} &= T(\frac{A}{y_{ss}})^{1/\alpha}. \end{aligned}$$

Dynamics. The Dynamics of the model are summarized in the following sketch. Let's substitute output per worker into the expression and obtain an expression for the change in population:

$$\begin{aligned} L_{t+1} - L_t &= \left(b_B A^{\beta_B} T^{\beta_B \alpha} (L_t)^{-\beta_B \alpha} - b_B A^{-\beta_D} T^{-\beta_D \alpha} (L_t)^{-\beta_D \alpha}\right) L_t \\ &= b_B A^{\beta_B} T^{\beta_B \alpha} L_t^{1-\beta_B \alpha} - b_B A^{-\beta_D} T^{-\beta_D \alpha} (L_t)^{1+\beta_D \alpha} \end{aligned}$$

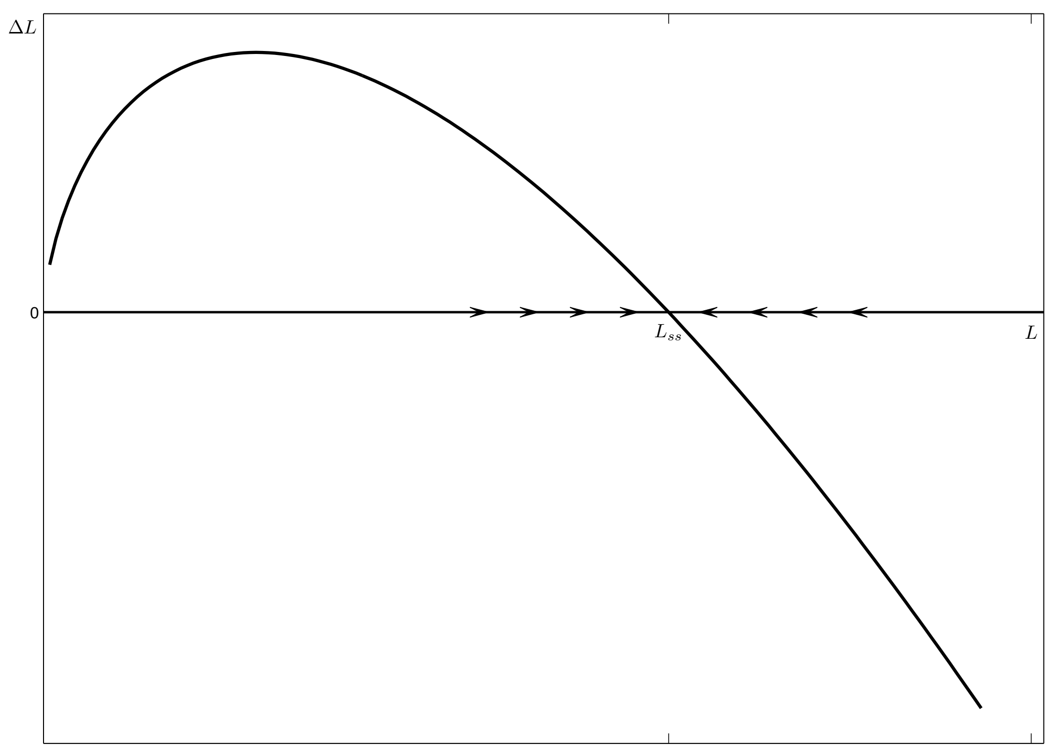

Figure 2: Malthus Mechanics

Figure 2 plots the change in population against the level of population. When population is below the steady state level, it is going to increase. When it is above the steady state, it is going to decrease.

One Increase in TFP. Let's compute GDP per capita, after an increase in productivity. What happens if productivity grows from A_o to A_{ss}. We obtain the following. At the moment of the change in TFP, there is an initial steady state L_0. Thus, we label it:

$$y_0 = A_0(T/L_0)^{\alpha}.$$

Then, there's a sudden increase in GDP per capita because TFP increases from A_o to A_{ss}. However, as GDP per capita increases, so does the growth rate of the population. And it will do so following the path we describe in Figure 3. Eventually, population will stabilize at:

$$L_{ss} = T\left(\frac{A_{ss}}{y_{ss}}\right)^{1/\alpha}.$$

However, since y_{ss} does not depend on the level of TFP, output per worker is the same. Along the transition, it will follow:

$$y_t = A_{ss}(T/L_t)^{\alpha} \text{ with intial condition } L_0 \text{ given.}$$

Figure 3 plots the transition path for population when the economy starts with a below steady state population. Population grows quickly initially, but the growth slows down as the economy approaches the steady state.

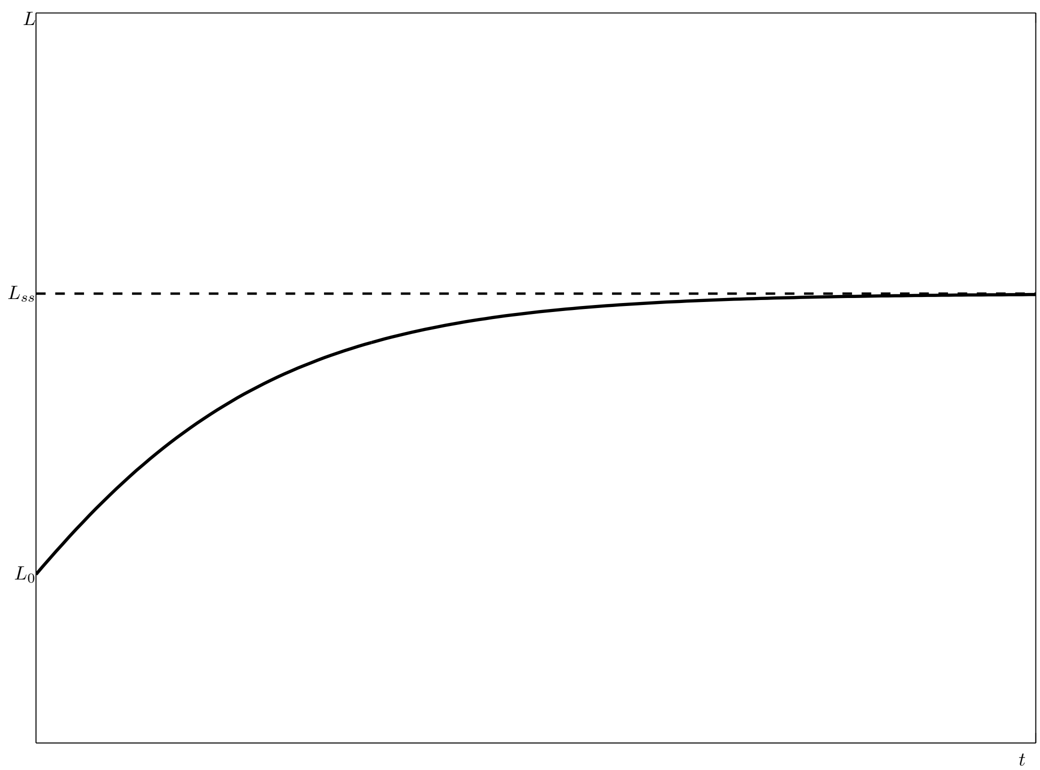

Figure 3: Malthus transition path

Since output is decreasing in L_t, output per worker begins to drop until it reaches the old steady state.

There are several conclusions obtained from the model: First, income per capita depends only on parameters of demography and not on the land endowment or technology. This is very important. Malthus's model predicts that if land is essential for the production of food, then changes in the technology levels will only have a medium run effect on total output per worker. From a technical standpoint, it means that if there's a non-reproducible factor, in this case land, sustained per-capita growth cannot be attained. Eventually, the increase in population will be such that, given the constraint of a fixed amount of land, the increase in population will exactly offset the increment in TFP in the calculation of output per worker. Of course, total output and population are growing.

The effects of a change in TFP are depicted in Figure 4 and 5. In Figure 5, shows that an increase in TFP will raise changes in population at any initial level. The economy will settle down at a higher a steady state. Figure 5, the transition from old steady to new steady state is depicted. Population start to grow immediately after the change in TFP. The growth eventually stops as the economy approaches the new steady state. During the process, output per capita will jump up and then decreases to the initial level.

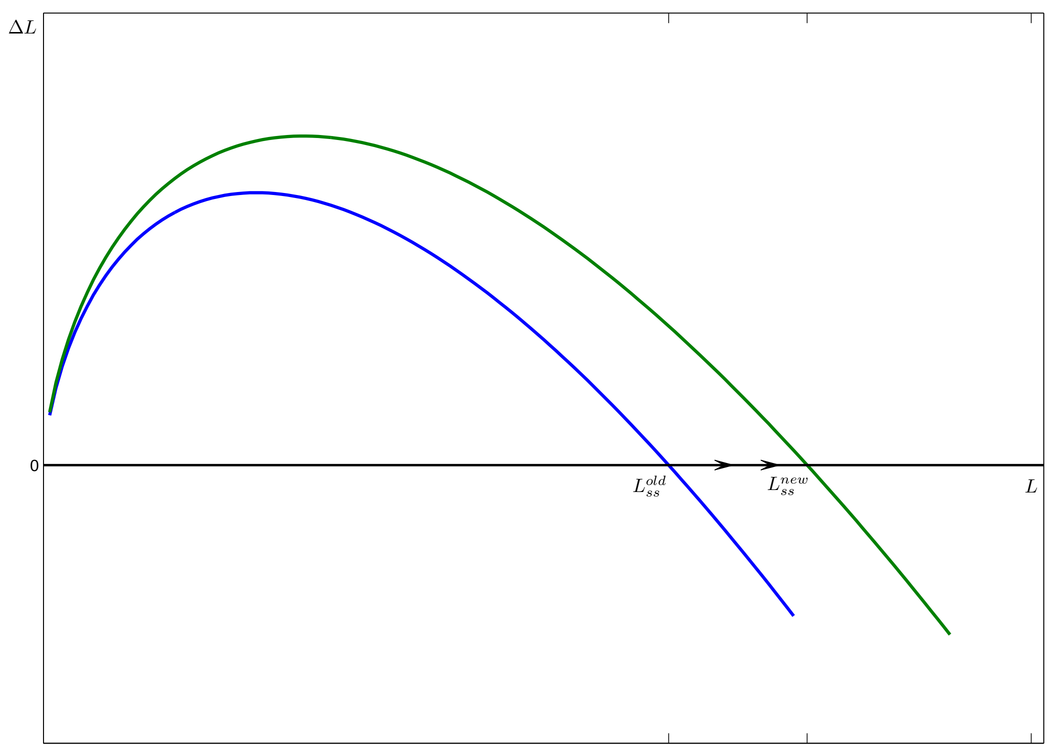

Figure 4: The effect of an increase in TFP

Second, the model does predict that epidemics raise income per capita. Some historical examples that document this pattern include the increase in the death rate —also known as the Black Death— in Europe that plagues north western Europe in the 1400th century. On the flip side, the model also predicts that increases in arable land —e.g., the settlement of North America— or improvements in

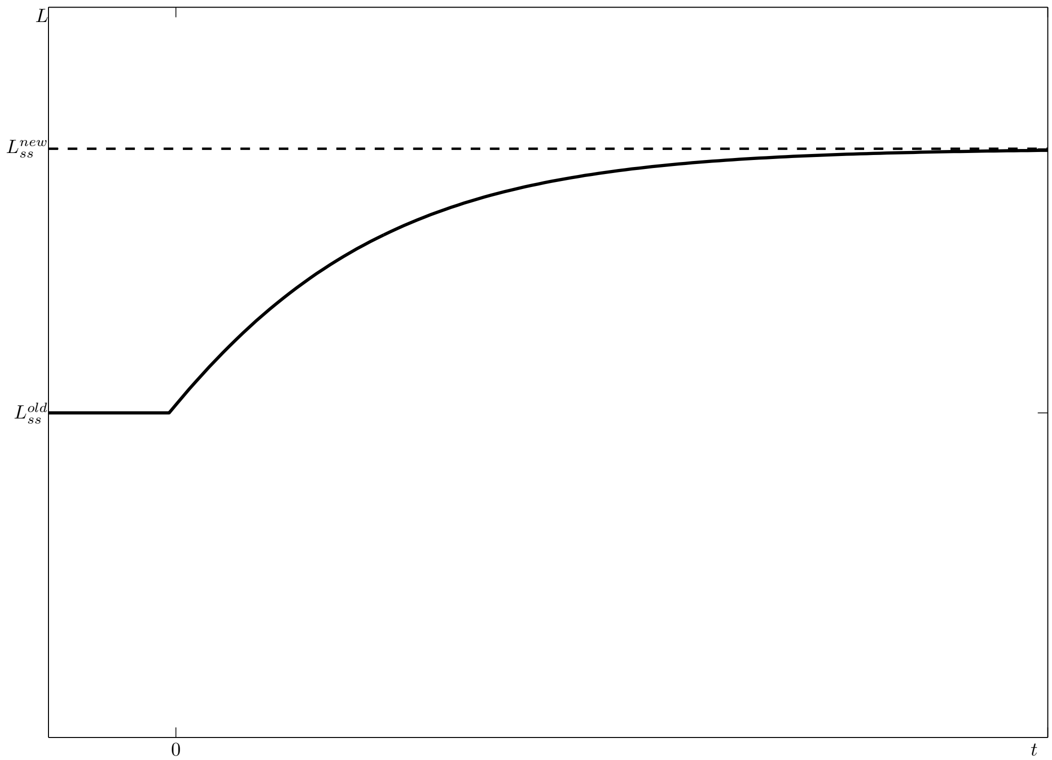

Figure 5: The effect of an increase in TFP

technology —the discovery of new harvests such as the potato— raise population, but not income per capita. Malthus collected evidence that living standards were very similar around the world up to 1800. He documented that real wages had been the same for a long time, that diets were the same for a long time —meat vs. starches— and that there was no evidence in increases in the average height of population as we do find after the industrial revolutions.

Nevertheless, there is evidence that north western Europe started to pull ahead after 1400. Why? Perhaps the Black Death played a substantial role in increasing death rates. There is evidence that shows lower fertility rates during the pre-industrial revolution and with the advent of the absolute states, there is evidence on less violence. More saving also had an influence in living standards.

Note that if we could expand the amount of land through investment we would be back in Solow's model so the key feature of the model is the diminishing returns to scale of labor given a fixed amount of land. Note also, that this model behaves as an alternative version of the poverty trap model that made savings an endogenous function of output per capita. To see this, we can add capital to the model, just as we did with the Solow model.

2.1 Malthusian Dynamics with Capital

What happens to the model when we alter the dynamics and include capital as in Solow's model? With capital, the production function becomes:

$$y_t = A(T/L_t)^\alpha k_t^\beta$$

and the accumulation of capital is:

$$k_{t+1} = sy_t + k_t(1-\delta)$$

Birth rates and death Rates are still the same**:**

$$L_{t+1} = \left(1 + b_B y_t^{\beta_B} - b_D y_t^{-\beta_D}\right) L_t$$

The steady state now is summarized by the following equations:

$$y^* = A(T/L^*)^\alpha k^{*\beta} \tag{1}$$

$$k^* = \frac{s}{\delta} y^* \tag{2}$$

and

$$y^* = (b_D/b_B)^{1/(\beta_B+\beta_D)}. \tag{3}$$

So replacing 3 in 2 we obtain:

$$y^* = A(T/L^*)^{\frac{\alpha}{1-\beta}} \left(\frac{s}{\delta}\right)^{\frac{\beta}{1-\beta}}$$

so clearing out this equation we obtain again the steady state labor supply:

$$L^* = T(A/y^*)^{\frac{1-\beta}{\alpha}} \left(\frac{s}{\delta}\right)^{\frac{\beta}{\alpha}}$$

so as we can see, the use of capital is not important in determining the outcomes of the model.

2.2 Malthusian Dynamics with Constant TFP growth

What happens if TFP increases consistently. Next, we show that there exists a balanced growth path with population growing at the same rate of TFP but GDP per capita remains fixed. TFP per capital is:

$$y_t = A_t(T/L_t)^\alpha$$

but now we let

$$\frac{A_{t+1}}{A_t} = (1+g).$$

If the conjecture is true, then,

$$y_{ss} = A_t (T/L_t)^{\alpha} = A_{t+1} (T/L_{t+1})^{\alpha}.$$

From here we conclude that:

$$\frac{L_{t+1}}{L_t} = (1+g)^{\frac{1}{\alpha}}.$$

Birth rates and death Rates are still the same**:**

$$L_{t+1} = \left(1 + b_B y_{ss}^{\beta_B} - b_D y_{ss}^{-\beta_D}\right) L_t$$

Thus, it must be true that:

$$\left(1 + b_B y_{ss}^{\beta_B} - b_D y_{ss}^{-\beta_D}\right) = (1+g)^{\frac{1}{\alpha}}$$

This equation has a unique solution, although we cannot obtain an analytic expression for it, we know it exists. The conclusion is that even if we add TFP growth, labor growth combined with a fixed factor (land) act like a dampening force.

3 Natural Resources

Natural Resources and Energy. The planet has a fixed stock of water, oil, land, and other energetic resources. Even if these resources are renewable, they may become a bottle neck for economic growth in the next hundred years. Malthus's predictions may become more relevant in the distant future if population continues to grow, and the stock of those resource doesn't catch up. We will return to the issue of natural resources in a future lecture when I will talk about the depletion of fish.

4 The Industrial Revolutions

Certainly the Industrial Revolution represents a counterfactual theory regarding output per person. Industrial revolutions represent a threshold episode in which mankind abandoned what seemed to be predominant Malthusian dynamics. I suggest reading David Landes's the Wealth and Poverty of Nations for a historical discussion. A more complicated question is why it happened in north west Europe and not China or Japan? Nevertheless, we can list some factors such as lower population growth, the role of institutions and rewards to invention as boosting factors that lead to solow dynamics. As we shall see later on, Jared Diamond sketched a powerful argument on why more technologically advanced societies were found in Europe or Asia thanks to a geographical advantage.

As we did with the Solow model, we are interested in using Malthus's model, it's failures to explain why the western world, and today developing countries have started to experience a substantial divergence. What explained the end of malthusian dynamics? There are a number of theories that explain that breakthrough.

5 From Malthus to Solow

Several theories have been developed in recent years. Many of them explain a tension between number of children and investment in human capital. I will discuss several of them. Matteo Cervellati and Uwe Sunde (AER 2005) explain a novel mechanism. They claim that as technology for increasing longevity improved, people had more incentives to invest in human capital reducing the fertility rates which in turn fostered greater increases in capital per worker and again higher returns to human capital fuelling the process again. The argument can be tracked back to Kremer and Chen who argued that the direct relation between income per capita and population growth rates are valid for unskilled workers but not for skilled workers.

Jeremy Greenwood, Ananth Seshadri, Guillaume Vandenbroucke (AER 2005) support the view that increases in technology provoked substantial increases in output per worker that eventually lead to an increase in the opportunity cost of having children. Nevertheless, the baby boom was explained by a sui generis episode in which household technology increased and allowed for more children.

Oded Galor provides a striking theory. His basic claim is that there was a self selection processes in technology that eventually allowed for the boost in human capital and progress towards a Neoclassical dynamics, regardless of the Malthusian dynamics.

Finally, Prescott and Parente (2005) build a model in which Malthusian Dynamics and Solow dynamics coexist in a model with 2 sectors. They show that the Malthusian sector, i.e. with a fixed land factor will always operate whereas the Solow model appears only if certain conditions for the rates of return occur. They cheat a little bit by assuming that population growth is a function of what model mechanics are operating more strongly.

6 Determinants of the Fertility Rate

Becker, Murphy, and Tamura (1990) study the determinants of fertility. Their model features Parent's decision about: Number of children and Education of children and their basic goal is to study how these decisions relates to economic growth.

The model as constructed over several simplifying assumptions: agents live for only two periods of life: Childhood and Adulthood. And only adults make decisions about: dividing time between work, time with children, and educating children and how many children to have. In that sense, the model endogeneizes fertility rates as opposed to the malthusian assumption that people behave, more or less like animals.

Preferences. The parents utility us V_t is the utility obtained from their own consumption:

$$V_t = u(c_t) + a(n_t)n_t V_{t+1}$$

with u an increasing and concave function of the parents utility and and a is an functional form for altruism which is assumed to be decreasing or constant.

Time Endowment. T is spent in to work hours, l_t, per child spent time v which is assumed fixed, and education per child h_t

$$T = l_t + n_t(v + h_t).$$

Human Capital, Income, and Education. Human capital is composed by a genetic endowment $\bar{H}$ and a stock H_t. Total human capital is obviously:

$$\bar{H} + H_t.$$

Income is obtained through a linear production function:

$$l_t(\bar{H} + H_t)$$

and the evolution of Human capital depends on the following formula:

$$H_{t+1} = Ah_t(\bar{H} + H_t)^{\beta}$$

so there are diminishing returns to education.

Consumption is shared among children according to the following formula:

$$c_t + n_t f = l_t(\bar{H} + H_t)$$

where f is spending per child.

Finally, altruism is defined as:

$$a(n_t) \equiv Cn_t^{-\varepsilon}.$$

6.1 Implications of the Becker, Murphy and Tamura Theory

(a) Spending time educating children is more worthwhile for parent who is herself educated. (b) A parent who is educated might find it more worthwhile to educate a child, and choose to spend more time doing so. (c) Taking time to educate a child is more costly the more children a parent has. (d) Thus a parent who decides to spend more time educating each child may choose to have fewer children.

6.2 Mathematical Outcome I

If H_t starts out low enough a parent may choose $h_t = 0$. Such a parent may choose a large n_t. Then by the next generation $H_{t+1} = 0$ from then and forever after. The economy stagnates

6.3 Outcome II

If H_t starts out high enough parents may keep adding to the human capital of their children. The stock of H_t grows over time. Parent will choose a smaller n_t. The analysis points to the complementarity between choosing a small number of children and educating them more intensively.