Lecture 3: Human Capital and Social Infrastructure Growth Theories

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"Once one starts to think about them (*questions about economic growth*) it is hard to think of anything else." Robert E. Lucas Jr.

1 Why doesn't capital flow from rich to poor countries?

This section is partially based on two of the most influential papers on growth of the nineties decade. I'm referring to Robert Lucas's 1988 JME paper "On the mechanics of economic development" and on his 1990 AER paper titled "Why doesn't Capital flow from rich to poor countries?" In previous lectures when presenting the neoclassical growth model, we overlooked an important piece of information.

Differences in capital per worker, under assumptions of a common technology, imply huge differences in the rates of return to physical capital in developed and poor countries. In Lucas's original work, he claimed that such differences would account for up to a 60-fold difference between the returns in the U.S. and India. Let's reconstruct his example by comparing the US and another country, Peru.

Recall from the first lecture the formula for per capita output under the neoclassical production function of diminishing returns to scale in capital-per-worker:

$$y_t = A_t k_t^{\alpha}. \tag{1}$$

Taking derivatives with respect to k_t, we obtain the marginal product of capital per worker. That is, the marginal return to an extra unit of capital. We thus have:

$$r_t = \alpha A_t k_t^{\alpha-1}$$

We can clear out k_t from (1) to obtain:

$$r_t = \alpha A_t^{1/\alpha} y_t^{\frac{\alpha-1}{\alpha}}. \tag{2}$$

Assuming that technology is the same and imposing a value of α close to 0.4, a 5 fold difference in output per worker in the U.S. and Peru imply a would imply that a difference in returns of

$$\frac{r_t^p}{r_t^{us}} = \left(\frac{1}{5}\right)^{-1.5} \simeq 11.2$$

This accounting exercise implies that the rate of return to capital per worker should be 11.2 times higher in Peru than in the US. Why then do American companies invest in the U.S. rather than in Peru? Replace the five-fold difference for a fifteen-fold difference and the numbers are just staggering! You obtain a 58 fold difference. The neoclassical model implies that there should be a huge flow of capital from rich to poor countries —but that doesn't happen.

Evidently, an answer to this question is that technology is different in both countries. However, you should ask, how come an American company cannot bring it's capital and it's technology from the US to Peru.

Clearly, there are many reasons why returns are not equalized among countries. One reason is that political instability may work as a detrimental factor for economic growth, but 12-fold figures can support any source of political risk.[1] Put a 50% expropriation risk and still, expected returns are 6 times higher. We can say the same thing about public goods such as roads and means of transportation, shipping costs etc.

The main focus of Lucas's paper were two features of technology. First, that part, but not all, technology is embedded in people, and this is represented in human capital. Second, that human capital leads to positive externalities that cannot be appropriated by workers. We explore these effects in the following subsections and various forms of human capital and its externalities and we discuss some evidence going forward.

2 Human Capital

In our first lecture, the Neoclassical model was characterized by 3 equations that boil down to a single one —the fundamental equation for capital accumulation. We can have an extension to that model that accounts for a broader version of labor, that takes into account human capital. We can call that factor, H_t. The neoclassical model is modified to obtain the following set of equations.

Aggregate Production. Again, output Y_t, is produced through some technological process F and three factors, physical capital K_t, human capital H_t, and effective units of labor A_tL_t. Here L_t denotes the population size with exogenous growth rate n, and A_t corresponds to the labor-augmented technology with exogenous growth rate g. We describe this by the equation:

$$Y_t = F\left(K_t, H_t, A_tL_t\right) \tag{3}$$

Notice that this production function is somewhat unusual, since it separates human capital H from

[1] Work by Amador and Aguiar (2006) presents a model with expropriation risk.

labor L as potential factors of production. We start from this form, since it is commonly used in the growth literature. We again specify a Cobb-Douglas functional form in terms of the units of effective labor:

$$F(K_t, H_t, A_tL_t) \equiv K_t^{\alpha} H_t^{\beta} (A_tL_t)^{1-\alpha-\beta}.$$

Capital Accumulation (Stock Equation). Again, as in the neoclassical growth model, capital evolves according to a stock equation. The stock equation simply summarizes the fact that capital tomorrow is today's capital minus a fraction δ_k that depreciates and today's investment I_t.

$$K_{t+1} = K_t - \delta_k K_t + I_t \tag{4}$$

Capital tomorrow will be used in future production. Following the Solow model we have learned, we assume the investment is proportional to total output, i.e. $I_t = s_k Y_t$.

Human Capital Accumulation (Stock Equation). Models differ on assumptions about whether human capital is independent of output or not. We adopt the assumption by Lucas in our model. Lucas (1988) shares the same spirit of his colleague at Chicago, Gary Becker, that human capital accumulation requires some time input taken out of the actual working labor force —talking about human capital spillover. Think of the labor resources lost when teachers educate children. More teachers means less output is going to be produced, but the benefit is that more human capital is accumulated. We could say the same thing of managers training employees.

To simplify the analysis, we will simply say that S_t is investment in human capital and that the resources employed are goods —again thinking of these as the forgone output lost to employing teachers in the production of human capital. The investment in human capital is assumed to be proportional to output as well, i.e. $S_t = s_h Y_t$. Like physical capital, human capital depreciates at a constant rate δ_h. Thus, we can write the equation of human capital accumulation as:

$$\text{Lucas Version: } H_{t+1} = s_h Y_t + (1 - \delta_h) H_t.$$

Aggregate Demand (Definition). Finally, we establish how production is distributed between consumption and whatever we invest in physical and human capital:

$$Y_t = C_t + I_t + S_t.$$

We now specify the model to discuss how things change with human capital differences.

2.1 Why doesn't capital flow from rich to poor countries?

Note that all that has changed from this setup and the Neoclassical setup is that we've added the term H_t to the production function. In this modified model, we can define human and physical capital per efficiency labor as

$$\hat{k}_t = \frac{K_t}{A_t L_t} \text{ and } \hat{h}_t = \frac{H_t}{A_t L_t}, \tag{5}$$

Tand the output per effective unit of labor can be written as

$$\hat{y}_t = \frac{Y_t}{A_t L_t} = \hat{k}_t^{\alpha} \hat{h}_t^{\beta}. \tag{6}$$

One of the findings of Lucas's paper was that once we take this broader version of human inputs, accounting for differences in human capital H_t, the differences in the rates of return are not that important. Lucas used calibrations by economist Anne Krueger —from the 60's. These studies suggested that human capital could be as much as five times greater in the US than in India, about the same with respect to Canada and other developed countries and that Israel's would be around 10% away from the US. What would these figures mean for the rates of return? The formula for the rates of return is given by

$$r_t = \alpha \frac{Y_t}{K_t} = \alpha \hat{k}_t^{\alpha-1} \hat{h}_t^{\beta}. \tag{7}$$

We can clear out $\hat{k}_t$ using (6) and obtain

$$r_t = \alpha \left(\hat{y}_t\right)^{\frac{\alpha-1}{\alpha}} \hat{h}_t^{\frac{\beta}{\alpha}}. \tag{8}$$

Because human capital is greater in the US than in developing countries, the ratio we had before turns out to be smaller. For example, if we assume $\beta = 0.3$,

$$\begin{aligned} \frac{r_t^p}{r_t^{us}} &= \left(\frac{\hat{y}_t^P}{\hat{y}_t^{US}}\right)^{-1.5} \left(\frac{\hat{h}_t^P}{\hat{h}_t^{US}}\right)^{0.75} \\ &= \left(\frac{y_t^P}{y_t^{US}}\right)^{-1.5} \left(\frac{h_t^P}{h_t^{US}}\right)^{0.75}, \end{aligned}$$

where $h_t = H_t/L_t$ denotes human capital per labor. Calibrating some numbers for Peru and the US again, assuming that output in the US 5 times grater, and human capital is also 25 times greater we obtain that the rates of return when considering human capital are about the same! To see this observe that:

$$\frac{r_t^p}{r_t^{us}} \simeq \left(\frac{1}{5}\right)^{-1.5} \left(\frac{1}{25}\right)^{0.75} = 1.$$

When applying this formula to India's numbers, the rates of return were different by a factor of 5. So for the Indian case, Lucas's claim was that the model was not enough. Note that we have not said anything about the accumulation of human capital yet, just said something about levels of human capital.

2.2 Steady-state Equilibrium

Now we proceed to discussing the steady-state equilibrium. Using the same steps as in the Solow model with population and technology growth, the laws of motion of $\hat{k}_t$ and $\hat{h}_t$ are

$$(1+n)(1+g)\hat{k}_{t+1} = s_k \hat{k}_t^{\alpha} \hat{h}_t^{\beta} + (1-\delta_k)\hat{k}_t,$$

$$(1+n)(1+g)\hat{h}_{t+1} = s_h \hat{k}_t^{\alpha} \hat{h}_t^{\beta} + (1-\delta_h)\hat{h}_t.$$

A steady-state equilibrium is now defined by human and physical capital per efficiency labor, $\left(\hat{k}_{ss}, \hat{h}_{ss}\right)$, satisfying the following two equations:

$$(1+n)(1+g)\hat{k}_{ss} = s_k \left(\hat{k}_{ss}\right)^{\alpha} \left(\hat{h}_{ss}\right)^{\beta} + (1-\delta_k)\hat{k}_{ss}$$

and

$$(1+n)(1+g)\hat{h}_{ss} = s_h \left(\hat{k}_{ss}\right)^{\alpha} \left(\hat{h}_{ss}\right)^{\beta} + (1-\delta_h)\hat{h}_{ss}.$$

The two-equation system has a unique pair of solution:

$$\hat{k}_{ss} = \left[\left(\frac{s_k}{n+g+ng+\delta_k}\right)^{1-\beta}\left(\frac{s_h}{n+g+ng+\delta_h}\right)^{\beta}\right]^{1/(1-\alpha-\beta)}, \tag{9}$$

$$\hat{h}_{ss} = \left[\left(\frac{s_k}{n+g+ng+\delta_k}\right)^{\alpha}\left(\frac{s_h}{n+g+ng+\delta_h}\right)^{1-\alpha}\right]^{1/(1-\alpha-\beta)}. \tag{10}$$

It shows that a higher saving rate in physical capital not only increases $\hat{k}_{ss}$ but also $\hat{h}_{ss}$. The same applies for a higher saving rate in human capital. This reflects the fact that the higher saving rate in physical capital, by increasing $\hat{k}_{ss}$, raises overall output and thus the amount invested in human capital (since s_h is constant). Given the solution (9) and (10), the output per effective unit of labor in steady state is

$$\hat{y}_{ss} = \left(\frac{s_k}{n+g+ng+\delta_k}\right)^{\alpha/(1-\alpha-\beta)}\left(\frac{s_h}{n+g+ng+\delta_h}\right)^{\beta/(1-\alpha-\beta)}. \tag{11}$$

Equation (11) shows that the relative contributions of the saving rates for physical and human capital on output per capita depend on the shares of physical and human capital–the larger is α, the more important is s_k and the larger is β, the more important is s_h.

2.3 Mankiw, Romer and Weil Regressions

Let's return to our analysis and test from the macroeconomic perspective if investments in human capital are important to explain cross country differences. This section borrows from the work of Mankiw, Romer and Weil (QJE 1992). To motivate regression analysis, they use a Solow model with physical and human capital accumulation which is very similar to our derivations from the previous section.

The First Regression.

The initial point of analysis is to consider the Solow model with only accumulation of physical capital, which we studied in previous lecture note. In fact, we can obtain it by setting $\beta = 0$:

$$\hat{y}_{ss} = \left(\frac{s_k}{n + g + ng + \delta_k} \right)^{\alpha/(1-\alpha)} \tag{12}$$

Recalling that $\hat{y}_{ss} = Y_t/(A_t L_t)$ and exogenous growth assumption $A_t = A_0(1+g)^t$, this implies the following regression equation which has to hold across countries:

$$\begin{aligned} \log\left(\frac{Y_t}{L_t}\right) &= \log\left(A_t\right) + t\log(1+g) \\ &+ \frac{\alpha}{1-\alpha}\log\left(s_k\right) - \frac{\alpha}{1-\alpha}\log\left(\delta_k + g + n + ng\right) \end{aligned}$$

To get exactly Mankiw, Romer and Weil's table, we invoke some of their assumptions: we assume that long-run growth rate of technology is constant across countries, therefore we can set $\log\left(A_t\right) + gt = a + \varepsilon$, and noticing that ng is negligible (this is due to the fact that they used continuous version of the model), $s = I/Y$, that $log\left(1+g\right) = g$ for g small we finally end up with their equation:

$$\log\left(\frac{Y_t}{L_t}\right) = a + \frac{\alpha}{1-\alpha}\log\left(\frac{I_t}{Y_t}\right) - \frac{\alpha}{1-\alpha}\log\left(\delta + g + n\right) + \varepsilon$$

After picking values for depreciation (also common across countries), all the rest of the variables in this equation ($\frac{Y_t}{L_t}, \frac{I_t}{Y_t}, n$) are observable and vary across countries. So Mankiw, Romer and Weil can run OLS regression in a cross-section of countries. There is a certain caveat to running this regression, which we will discuss later.

Table 1 shows the results of OLS estimation. Two main points are worth emphasizing.

First, note that the regression gets the signs at $\log s_k$ and $\log(n+g+\delta)$, in line with what theory predicts. Moreover, not only signs are correct, but they are very close in magnitude (1.42 and -1.97 in the baseline estimation, first column of table 1), which is what we expect from Solow model.

Second, notice that coefficient at saving rate is equal to 1.42. This implies

$$\frac{\alpha}{1-\alpha} = 1.42 \Rightarrow \alpha = \frac{1.42}{1+1.42} \approx 0.59$$

But recall from our previous lecture that α also corresponds to the share of capital in national income and therefore should be $\alpha = 0.4$, which is much lower that 0.59. In other words, the regression analysis predicts that the saving rate matters much more for the output per capita differences than what Solow model tells us. Based on this, Mankiw, Romer and Weil motivate augmenting Solow model with Human capital.

Table 1: Estimates of the basic Solow model

	MRW	Updated data	
	1985	1985	2000
$\log(s^k)$	1.42	1.01	1.22
	(.14)	(.11)	(.13)
$\log(n+g+\delta)$	-1.97	-1.12	-1.31
	(.56)	(.55)	(.36)
Adjusted R^2	.59	.49	.49
Implied α	.59	.50	.55
Number of observations	98	98	107

Note: Standard errors are in parentheses. Computations taken from Acemoglu (2009).

The Augmented Regression.

Here we follow a slightly modified version of the model which we solved in the previous section. Taking logs of equation (11), assuming that $\delta_k = \delta_h = \delta$, and making the same assumptions as before, we can obtain the following regression equation :

$$\log\left(\frac{Y_t}{L_t}\right) = a + \frac{\alpha}{1-\alpha-\beta}\log\left(\frac{I_t}{Y_t}\right) + \tag{13}$$

$$\frac{\beta}{1-\alpha-\beta}\log\left(s^h\right) - \frac{\alpha+\beta}{1-\alpha-\beta}\log\left(\delta+g+n\right) + \varepsilon \tag{14}$$

Notice that with this setup, investments in physical capital, increase, through an interaction effect, the human capital and therefore the model predicts, higher income levels through this mechanism. Mankiw, Romer and Weil (1992) clear out from the steady state level of s^h, then we are able to express this last equation in a version closer to the one we had in the previous section:

$$\log\left(\frac{Y_t}{L_t}\right) = a + \frac{\alpha}{1-\alpha}\log\left(\frac{I_t}{Y_t}\right) + $$
$$-\frac{\alpha}{1-\alpha}\log\left(\delta+n+g\right) + \frac{\beta}{1-\alpha}\log\left(\hat{h}_{ss}\right) + \varepsilon$$

This equation is identical to the equation obtained before, except that in the previous section where we did not include the positive externality implied by human capital. The main contribution of the paper is that omitting this term will bias the estimates in Table 1. This is important because, as we noted before, the regression in table 1 was overstating the results.

For estimation, go back to equation (14). Mankiw, Romer and Weil use share of working age population that went to secondary school as a proxy to measure s_h. As we can see below, the regression fixes the performance of the model: the implied α is much closer to what we got from Growth Accounting than in Table 1. On top of that, notice that R-squared is equal to 78%, which basically means that physical and human capital accumulation can explain up to 78% of the variation of income differences between countries.

Table 2: Estimates of the augmented Solow model

	MRW	Updated data	
	1985	1985	2000
$\log(s^k)$	.69	.65	.96
	(.13)	(.11)	(.13)
$\log(n+g+\delta)$	-1.73	-1.02	-1.06
	(.41)	(.45)	(.33)
$\log(s^h)$	.66	.47	.70
	(.07)	(.07)	(.13)
Adjusted R^2	.78	.65	.36
Implied α	.30	.31	.36
Number of observations	98	98	107

Note: Standard errors are in parentheses. Computations taken from Acemoglu (2014).

Caveats of the approach by Mankiw, Romer and Weil

Mankiw, Romer and Weil make a nice argument about how Solow model augmented with human capital can explain a lot of cross-country income differences, but the OLS estimation approach undertaken by Mankiw, Romer and Weil may be problematic for the following reasons.

First, for OLS estimates to be valid, they have to assume that saving rate s_k and population growth rate n are independent from error term ϵ, which by construction incorporates technology differences between countries. It is very unlikely that the differences in technology do not matter for country's saving rate and population growth rate. While this is ok for basic macroeconomic model, it is not acceptable for econometric estimation. This will make estimates of α biased.

Second and probably more important, the OLS regression when the authors add human capital shows strong association of output per capita with education, but it does not prove that education *causes* income differences. It might very well be the case that citizens of richer countries choose to spend more time and resources on education because they are already rich. In the data we will see positive association between education and income per capita, but the reason for that positive association would be exactly the opposite of Mankiw, Romer and Weil's interpretation.

One way to overcome this is Instrumental Variables approach, but this is beyond the scope of this class. We note however that in practice it is hard to find a good and convincing instrumental variable, which is why even influential papers like Mankiw, Romer and Weil; have to resort to simple OLS approach.