1 Man versus Machine

So far we have studied models in which technology contributes to the welfare of workers by enhancing their salaries. However, a common theme in history has been a constant conflict between the development of new technologies and wages. Some technologies are beneficial for workers. Other ones hurt them because they allow machines to compete with workers. So now let's think of this problem.

1.1 The Decline in the Labor Share

Recent work by Neiman and Karabarbounis (2014) has demonstrated that one of the Kaldor facts, a constant labor share has begun to weaken in recent decades. The decline in the labor share appears to be a global phenomenon that shows up in a large number of countries with different characteristics. For example, about 42 countries out of a sample of 59 countries studied by Neiman and Karabarbounis (2014) showed a downward trend in labor share between 1975 and 2012. Figure 1 shows the global trend of labor share since 1975. The labor share for the corporate sector has declined steadily about 6 percentage points. A similar trend is found for the overall economy. Figure 2 shows that the trend of labor share for all the countries in their sample. A declining labor share is found in many countries. In particular, it is observed in most of the large economies emphasized in the figure, with UK as the only exception. The decline in China is worth mentioning as it is a developing country, which is predicted by some theories to have a rising labor share due to the export of labor intensive goods. Neiman and Karabarbounis (2014) attribute this new fact to a combination of a drop in the cost of producing capital combined with type of production function that differs from the one we studied so far, the Cobb-Douglas.

Next we will study a model related to recent work by Acemoglu and Restrepo. I present it next.

1.2 A Model with Competing Capital

Suppose that there are two types of capital. The first type of capital is K_t. It is the same type of capital that we have studied so far. In addition, there's the Z_t capital. I call it Androids, and denote it Z_t from the first letter of the word Zombie —the zombies are here to hunt us.

Both forms of capital are accumulated via:

$$K_t = s_k Y_t \text{ and } Z_t = s_z Y_t.$$

This is the same formula we studied before, except that now, depreciation is set to 1 for simplicity.

Let's assume that there is a single firm. Output is given by:

$$Y_t = K_t^{\alpha} \left(A\left(Z_t + L_t\right)\right)^{1-\alpha}$$

Figure 1: The global decline of labor share

Note: This figure comes from Figure 1 of Neiman and Karabarbounis (2014). The figure shows year fixed effects from a regression of corporate and overall labor shares that also include country fixed effects to account for entry and exit during the sample. The regressions are weighted by corporate gross value added and GDP measured in U.S. dollars at market exchange rates. They normalize the fixed effects to equal the level of the global labor share in their dataset in 1975.

where A is labor (androids) enhancing productivity.

Consider now the value of output per worker. It is given by:

$$y_t = \frac{Y_t}{L_t} = k_t^{\alpha} \left(A \left(z_t + 1 \right) \right)^{1-\alpha}.$$

where z_t is the number of zombies per worker.

From the capital accumulation equation we obtain:

$$k_t = s_k y_t \text{ and } z_t = s_z y_t.$$

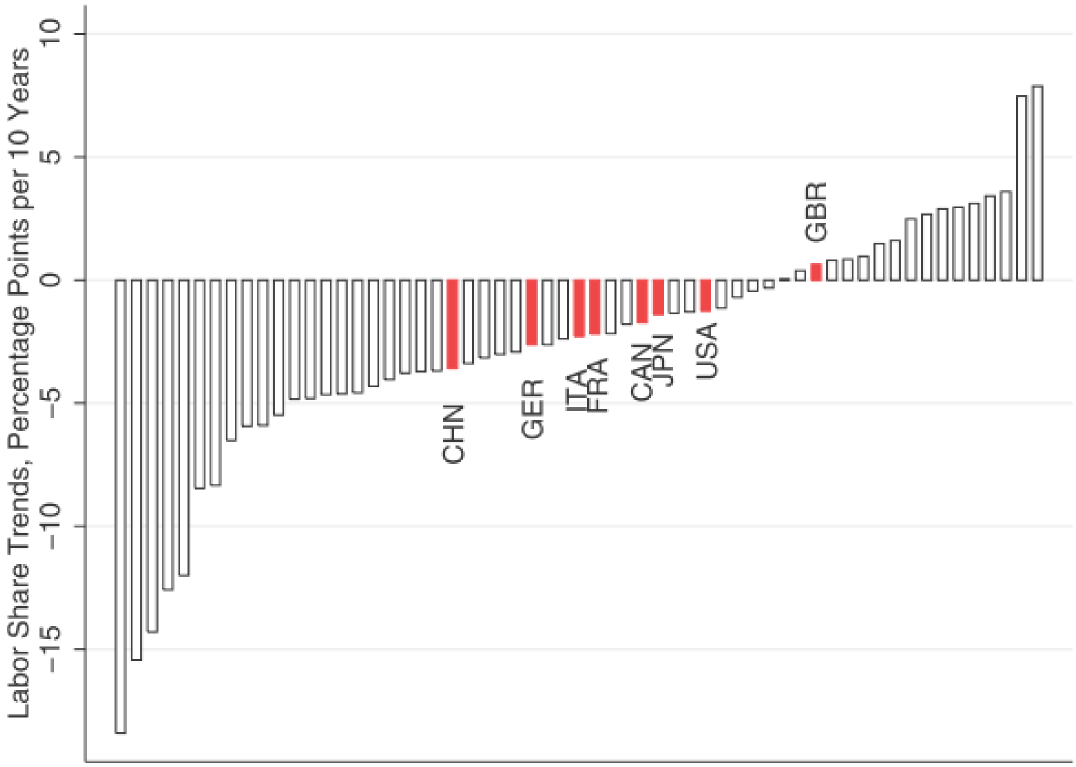

Figure 2: The trend of labor share in all countries

Note: This figure comes from Figure 3 of Neiman and Karabarbounis (2014). The figure shows estimated trends in the labor share for all countries in their data set with at least 15 years of data starting in 1975. Trend coefficients are reported in units per 10 years (i.e., a value of 5 means a 5 percentage point decline every 10 years). The largest eight economies are shaded.

Thus, output per worker is given by:

$$\begin{aligned}
y_t &= y_t^{\alpha} s_k^{\alpha} \left(A\left(s_z y_t + 1\right)\right)^{1-\alpha} \rightarrow \\
y_t^{1-\alpha} &= s_k^{\alpha} \left(A\left(s_z y_t + 1\right)\right)^{1-\alpha} \rightarrow \\
y_t &= s_k^{\frac{\alpha}{1-\alpha}} A\left(s_z y_t + 1\right) \rightarrow \\
y_t &= \frac{A s_k^{\frac{\alpha}{1-\alpha}}}{1 - A s_z s_k^{\frac{\alpha}{1-\alpha}}}.
\end{aligned}$$

Steady State. What happens if $1 < A s_z s_k^{\frac{\alpha}{1-\alpha}}$? In this case, the economy doesn't have a steady state because it continues to grow forever. In particular, note that

$$\begin{aligned} y_{ss} &= \frac{As_k^{\frac{\alpha}{1-\alpha}}}{1 - As_z s_k^{\frac{\alpha}{1-\alpha}}} \\ &= \frac{1}{A^{-1}s_k^{-\frac{\alpha}{1-\alpha}} - s_z} \end{aligned}$$

implies that $1/s_z > As_k^{\frac{\alpha}{1-\alpha}}$. The problem is that if s_z is too high, this economy, begins accumulating workers and machines and never stops growing. Thus, to make the problem interesting, let's assume that the condition holds.

Labor Share. Let's compute wages. From the firm's first order condition we obtain:

$$\begin{aligned} w_t (L_t + Z_t) &= (1-\alpha) K_t^{\alpha} (A (Z_t + L_t))^{1-\alpha} \\ &= (1-\alpha) y_{ss} L_t. \end{aligned}$$

Thus, we obtain the following condition:

$$w_t (L_t + s_z y_{ss} L_t) = (1-\alpha) y_{ss} L_t.$$

If we divide both side by L_t we obtain the following:

$$\begin{aligned} w_t &= (1-\alpha) \frac{y_{ss}}{(1 + s_z y_{ss})} \\ &= (1-\alpha) \frac{\frac{1}{A^{-1}s_k^{-\frac{\alpha}{1-\alpha}} - s_z}}{1 + \frac{s_z}{A^{-1}s_k^{-\frac{\alpha}{1-\alpha}} - s_z}} \\ &= (1-\alpha) \frac{\frac{1}{A^{-1}s_k^{-\frac{\alpha}{1-\alpha}} - s_z}}{\frac{A^{-1}s_k^{-\frac{\alpha}{1-\alpha}} - s_z + s_z}{A^{-1}s_k^{-\frac{\alpha}{1-\alpha}} - s_z}} \\ &= \frac{(1-\alpha)}{A^{-1}s_k^{-\frac{\alpha}{1-\alpha}}}. \end{aligned}$$

As you can observe, the steady state level of wages does not depend on the investment in androids. Androids increase total output. However, they don't translate into higher wages. Rather, they translate into higher rental rate for capital —both of physical capital and androids.

This aspect already shows that this simple model is capable of generating a widening of the income distribution, whereas it doesn't generate necessarily a decline in wages. The reason is that, although the greater competition from androids increases, the capital stock also increases.

Let's see why. Suppose we increase s_z a little bit. Then, we know at that

$$\frac{\partial y_{ss}}{\partial s_z} = -\frac{1}{\left(A^{-1} s_k^{-\frac{\alpha}{1-\alpha}} - s_z\right)^2} (-1) = y_{ss}^2 > 0$$

Thus, total output increases by:

$$\frac{\partial Y_{ss}}{\partial s_z} = \frac{\partial y_{ss}}{\partial s_z} L = y_{ss} Y_t.$$

Since total output increases but wages don't increase, it means that

$$\begin{aligned} w_t \frac{\partial L_t}{\partial s_z} + w_t \frac{\partial s_z}{\partial s_z} y_{ss} L_t + w_t s_z \frac{\partial y_{ss}}{\partial s_z} L_t &= (1-\alpha) \frac{\partial y_{ss}}{\partial s_z} L_t \rightarrow \\ w_t Y_{ss} + w_t s_z y_{ss} Y_{ss} &= (1-\alpha) y_{ss} Y_{ss} \\ w_t (1 + z_{ss}) &= (1-\alpha) y_{ss}. \end{aligned}$$

Which is verified. So what happens is that as the number of Androids increase, wages don't change for two effects. First, there is more competition. However, at the same time, the capital that complements labor increases. The two effects exactly offset each other. Since the labor force doesn't change, this entails that androids and physical capital earn the same amounts.