Fall 2017 Econ 164 Mid-term Exam

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November 13, 2017

The exam has 4 questions for 100 points in total. You have 60 minutes to finish the exam.

NOTE THAT POINTS VARY BETWEEN QUESTIONS.

DO NOT OPEN UNTIL THE EXAM BEGINS

Name:

UID:

Grade:

Table 1: Growth Accounting: U.S. and Europe: 1973-2011

Country	Growth Rate of GDP	Contribution from Capital	Contribution from Labor	TFP Growth Rate
Germany	0.0188	0.0081	0.0066	0.0041
($\alpha = 0.33$)		(43%)	(35%)	(22%)
Italy	0.0181	0.0118	0.0063	0.0001
($\alpha = 0.41$)		(65%)	(35%)	(0.4%)
Spain	0.0264	0.0151	0.0085	0.0031
($\alpha = 0.36$)		(57%)	(31%)	(12%)
U.K.	0.0202	0.0086	0.0018	0.0097
($\alpha = 0.36$)		(43%)	(9.0%)	(48%)
U.S.	0.0263	0.0090	0.0092	0.0081
($\alpha = 0.35$)		(34%)	(35%)	(31%)

Data Source: Penn World Tables 8.1.

1. Consider the Solow model with no population growth, but with technological progress. Suppose that at time $t = 0$ the number of workers in the economy drops because a portion of workers decide to retire from the workforce. This is what happened in Europe since the 1970s when many countries adopted strict labor regulations. Figure 1 shows an index of the labor force in Europe since that period. You can see that there was a decline in labor force participation (the number of people working). Answer the following questions in the light of this model. (40 points)

a. What is the long-run effect (that is, the effect in the balanced growth path) of the decrease in the labor force on the level of output per worker and its growth rate? (10 points)

b. What is short-run effect on total output? Draw a picture that plots the expected behavior of the natural logarithm of output against time on the x-axis. Note: the x-axis should start at $t = 0$, include $t = T$ as the date of the reform and end at a date such that output has converged to the balanced growth path. (10 points)

c. Over the 1950's and until the 1970's, Europe was catching up with the US. Since the 70's, the European countries stopped converging to the US and began departing. Explain how you could use the Solow model to decompose how much of the slowdown in Europe has to do with labor participation. (10 points)

d. Now suppose that these labor regulations lead to "shirking" so that workers report more hours than they actually work. Given the reported data, explain how this would be picked up by TFP in the growth accounting exercise presented in Table 1. (10 points)

Answers:

a. There would be no effect. In the long run we would reach the new balanced growth path, where output per effective worker grows at the same rate of technological growth.

b. Output will drop in the initial period because of the reduction in the workforce. In the same period the capital per efficiency unit of work has to go above its steady state value. In order to go back to the balanced growth path, the economy will divest some capital (i.e. reduce the amount of total capital) in the short term. Therefore, in the periods following the decline in the workforce, total output will grow at a smaller rate than the rate of technological growth. Over time, output growth will approach the growth rate of technology.

Figure: Labour force in Europe: 1965-1980 (Index 1965=100)

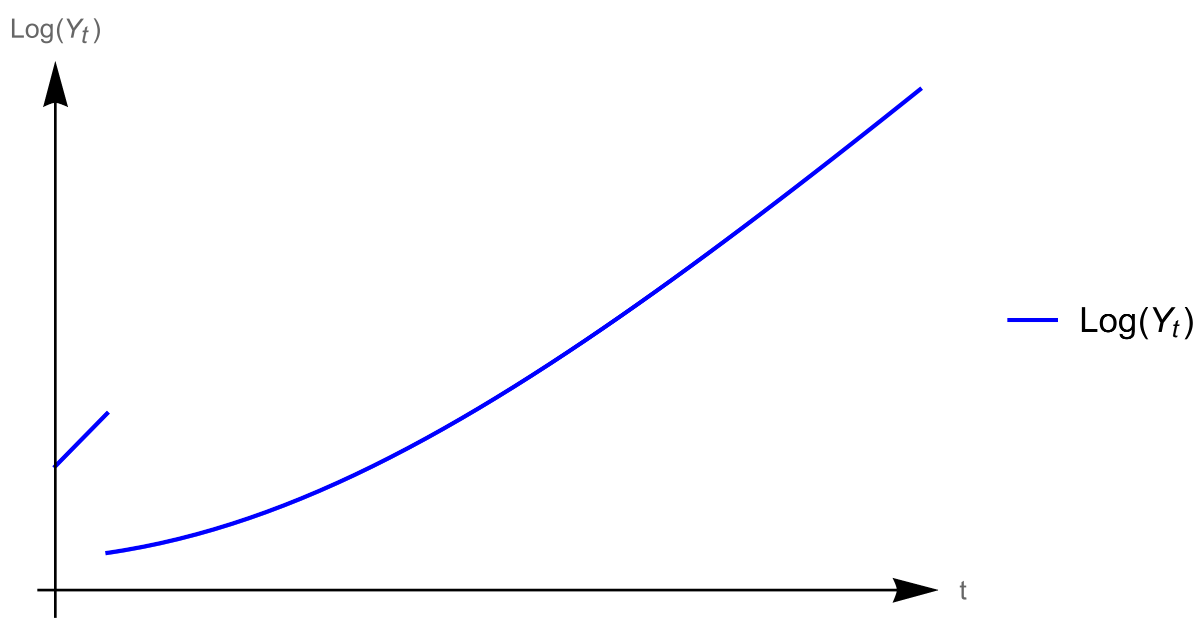

Figure 2: Log Output

c. According to the Solow decomposition Δ. Using the model with no population growth and technological process, we would conclude that labor had no role in growth (since Δ) and that $\Delta\%$A and $\Delta\%$K grow the at constant rate of technology $\tilde{x}$. So the trend growth would be $\tilde{x}$. Following a reduction in the labor force (Δ), we would obtain growth rates below the trend. In particular, the growth rate would decline by an amount equal to the percentage decrease in the labor force multiplied by the labor share $1-\alpha$.

d. Since workers report more hours than they actually work, the growth rate calculated in data is higher than the actual growth of labor input. Recall that from the Solow Decomposition we have

$$\Delta\%\hat{\tilde{A}}_t = \Delta \tag{1}$$

Where $\hat{\tilde{A}}$ is the TFP estimated. Assuming no measure error in GDP, capital or α, we have

$$\Delta\%\tilde{A}_t - \Delta\%\hat{\tilde{A}}_t = -(1-\alpha)\left(\Delta\%L_t - \Delta\%L_t^{data}\right) \tag{2}$$

Since $\Delta\%L_t^{data} > \Delta\%L_t$, it is straightforward to see that $\Delta\%\tilde{A}_t > \Delta\%\hat{\tilde{A}}_t$. This means that the TFP is underestimated in our calculations. This might be a potential reason why the estimated TFP in table 1 is very close to zero for European countries.

2. We have learned that capital per efficiency units of labor in the Solow model with no population growth and with technology growing at rate g is given by the following equation:

$$\tilde{k}_t = \left[\frac{\mathrm{s}}{\delta+g} + \left(\tilde{k}_0^{1-\alpha} - \frac{\mathrm{s}}{\delta+g}\right)\exp\left(-\left(1-\alpha\right)\left(\delta+g\right)t\right)\right]^{\frac{1}{1-\alpha}}.$$

Answer the following questions using this equation. Use the following standard numbers for the Solow Model when necessary: $s = 0.15$, $\alpha = 0.33$, g=0.025 and $\delta = 0.08$. (20 points)

a. From this equation, find the steady state $\tilde{k}_{ss}$ for capital per efficiency units of labor. (5 points)

For questions (b) and (c), suppose that a country has just finished a war at time $t = 0$. During the war, the government had imposed strict rationing and used the extra resources to build up industrial capacity to help the war effort. Once the war is over, the government allows savings to return to its normal level but the capital stock is high, so that $\tilde{k}_0 = 2\tilde{k}_{ss}$.

b. Find an expression (in terms of the parameters of the model) for the time it will take for $\tilde{k}_t$ to be 1% **above** its steady state. (10 points)

c. Find an exact numeric value for the time needed for $\tilde{k}_t$ to reach 1% above its state state. (5 points)

Answers:

a. From last equation,

$$\lim_{t\to\infty}\tilde{k}_t^{1-\alpha} = \frac{\mathrm{s}}{\delta+g} + \lim_{t\to\infty}\left(\tilde{k}_0^{1-\alpha} - \frac{\mathrm{s}}{\delta+g}\right)\exp\left(-\left(1-\alpha\right)\left(\delta+g\right)t\right) \tag{3}$$

Assuming $\tilde{k}_0 > 0$,

$$\lim_{t\to\infty}\left(\tilde{k}_0^{1-\alpha} - \frac{\mathrm{s}}{\delta+g}\right)\exp\left(-\left(1-\alpha\right)\left(\delta+g\right)t\right) = 0$$

Hence

$$\lim_{t\to\infty}\tilde{k}_t = \left(\frac{\mathrm{s}}{\delta+g}\right)^{\frac{1}{1-\alpha}}$$

b. We want $\tilde{k}_t = 1.01\tilde{k}_{ss}$, therefore

$$1.01^{1-\alpha}\left(\frac{s}{\delta+g}\right) = \frac{\mathrm{s}}{\delta+g} + \left(2^{1-\alpha}\left(\frac{s}{\delta+g}\right) - \frac{\mathrm{s}}{\delta+g}\right)\exp\left(-\left(1-\alpha\right)\left(\delta+g\right)t\right)$$

Which simplifies to

$$1.01^{1-\alpha} - 1 = \left(2^{1-\alpha} - 1\right)\exp\left(-\left(1-\alpha\right)\left(\delta+g\right)t\right)$$

Divide and take the log

$$\log\left(\frac{1.01^{1-\alpha}-1}{2^{1-\alpha}-1}\right) = -\left(1-\alpha\right)\left(\delta+g\right)t$$

Thus

$$t = -\frac{1}{\left(1-\alpha\right)\left(\delta+g\right)}\log\left(\frac{1.01^{1-\alpha}-1}{2^{1-\alpha}-1}\right)$$

c. Replace the numbers in the previous formula to get $t \simeq 61.95$

3. Let's consider the Solow model with population and technology growth. The steady state for capital per effective labor $\tilde{k}_{ss}$ is given by

$$\tilde{k}_{ss} = \left(\frac{s}{\delta + n + \tilde{x} + n\tilde{x}}\right)^{\frac{1}{1-\alpha}},$$

Answer the following questions. (20 points)

a. How will the steady state value $\hat{k}_{ss}$ change in response to a 50% decrease in the saving rate? Let the old and new steady state values be $\tilde{k}^o_{ss}$ and $\tilde{k}^n_{ss}$ respectively, and describe the relationship between them. (5 points)

b. Let's assume $\alpha = 0.3$, $s = 0.15$, $\delta = 0.05$, $n = 0.01$, and $\tilde{x} = 0.01$. Find an exact value for capital in the steady state. (5 points)

c. If the depreciation rate is increased to 0.08, what is capital in the new steady state? Is it higher or lower than in the original steady state? Provide some intuition for your result. (10 points)

a. $k^o_{ss} = \left(\frac{s}{\delta+n+\tilde{x}+n\tilde{x}}\right)^{\frac{1}{1-\alpha}}$ while $k^n_s = \left(\frac{0.5s}{\delta+n+\tilde{x}+n\tilde{x}}\right)^{\frac{1}{1-\alpha}}$ giving us that $\frac{k^n_{ss}}{k^o_{ss}} = (0.5)^{\frac{1}{1-\alpha}}$

b. This is just plugging in correctly for the given values. This gives us:

$$k^o_{ss} = \left(\frac{0.15}{0.05 + 0.01 + 0.01 + 0.0001}\right)^{\frac{1}{0.7}} \approx 2.96457$$

$$k^n_{ss} = \left(\frac{0.075}{0.05 + 0.01 + 0.01 + 0.0001}\right)^{\frac{1}{0.7}} \approx 1.10133$$

c. Raising the depreciation rate means that, at any given level of capital, more capital will disappear each period. Thus, to maintain a given level of capital we need to have a higher $\frac{f(k)}{k}$. In words, this corresponds to a higher average (and thus marginal) product of capital, which will balance out the higher depreciation rate. We know that $\frac{f(k)}{k}$ is decreasing in k given our assumption about $f(k)$ so we will need to have a lower k to achieve this result. Graphically, this corresponds to the intersection of gross investment and augmented depreciation intersecting lower on the (concave) $sf(k)$ curve given an upward shift in the augmented slope of the depreciation schedule.

$$k^o_{ss} = \left(\frac{0.15}{0.08 + 0.01 + 0.01 + 0.0001}\right)^{\frac{1}{0.7}} \approx 1.78213$$

$$k^n_{ss} = \left(\frac{0.075}{0.08 + 0.01 + 0.01 + 0.0001}\right)^{\frac{1}{0.7}} \approx 0.66206$$

4. Good and Bad Governments. (20 points)
Consider a version of the Solow model with a Government, which can be either Good or Bad. The law of motion for aggregate capital in the Solow model is given by

$$K_{t+1} = I_t + (1-\delta)K_t,$$

where I_t is total investment in this economy. Define I_t^{prv} and I_t^G as the private investment and government investment, respectively. Of course,

$$I_t = I_t^{prv} + I_t^G \tag{4}$$

We assume that private investment, I^{prv}, is a constant proportion of disposable income:

$$I_t^{prv} = sY_t^D$$

and Y_t^D is disposable income, given by:

$$Y_t^D = Y_t - G_t$$

where we assume that $G_t = \tau Y_t$, with $\tau \in (0,1)$ a parameter.

a. Provide an expression for Investment only as a function of total income. (2 points)
b. Use your equation for Investment to write the law of motion for capital per worker. In this case, the government is bad and tosses G_t into the ocean. That is, $I_t^G = 0$. What is GDP per capita in the steady state? (3 points)
c. Now suppose we have a good government, and that G_t is invested into capital. That is, $I_t^G = G_t$. Provide a new expression for the law of motion for capital, again in terms of Y_t and other parameters of the model. Now what is the steady-state level of GDP per capita? (5 points)
d. Let consumption be given by $C_t = (1-s)Y_t^D$. Suppose that a good government can't control private savings (s), but it can control its own level of taxing and spending (τ). What is the optimal choice of τ? Hint: it could be useful to remember the Golden rule that we derived in class. (10 points)

a. As announced in class, the question should say PRIVATE investment, not total investment. $I^{prv} = sY_t^D = s(Y_t - G_t) = s(Yt - \tau Y_t) = s(1-\tau)Y_t$. Total investment would just be this plus government investment, but we do not yet know what the government chooses for I_t^G.

b. Any assumption about growth of technology or labor is fine. Here we allow for them to grow, but most students on the exam (as announced in class) assumed no growth in technology or labor.

$$\begin{aligned}
K_{t+1} &= I_t^{prv} + I_t^G + (1-\delta)K_t \\
K_{t+1} &= s(1-\tau)Y_t + 0 + (1-\delta)K_t \\
(1+\tilde{x})(1+n)\frac{K_{t+1}}{\tilde{A}_{t+1}L_{t+1}} &= s(1-\tau)\frac{Y_t}{\tilde{A}_tL_t} + (1-\delta)\frac{K_t}{\tilde{A}_tL_t} \\
(1+\tilde{x})(1+n)k_{t+1} &= s(1-\tau)y_t + (1-\delta)k_t
\end{aligned}$$

In the steady state, we know that $k_{t+1} = k_t = k_{ss}$ so we have:

$$\begin{aligned}
(1+\tilde{x})(1+n)k_{ss} &= s(1-\tau)y_{ss} + (1-\delta)k_{ss} \\
\frac{(\delta+\tilde{x}+n+n\tilde{x})}{s(1-\tau)}k_{ss} &= y_{ss}
\end{aligned}$$

This equation is correct but it doesn't fully answer the question since we know that y_{ss} and k_{ss} are (potentially) functions of each other. Thus, we now make the standard assumption from the Solow model that $y_{ss} = k_{ss}^{\alpha}$ in order to keep making progress, giving us:

$$\begin{aligned}
\frac{(\delta + \tilde{x} + n + n\tilde{x})}{s(1-\tau)} k_{ss} &= k_{ss}^{\alpha} \\
k_{ss} &= \left(\frac{s(1-\tau)}{\delta + \tilde{x} + n + n\tilde{x}} \right)^{\frac{1}{1-\alpha}} \\
\frac{Y_{t,bg}}{L_{t,bg}} &= \tilde{A}_{t,bg} \left(\frac{s(1-\tau)}{\delta + \tilde{x} + n + n\tilde{x}} \right)^{\frac{\alpha}{1-\alpha}}
\end{aligned}$$

c. This question is almost the same as the previous question, except now we have $I_t^G = \tau Y_t$ instead of being zero. This gives us:

$$\begin{aligned}
K_{t+1} &= I_t^{prv} + I_t^G + (1-\delta)K_t \\
K_{t+1} &= s(1-\tau)Y_t + \tau Y_t + (1-\delta)K_t \\
K_{t+1} &= [s + \tau(1-s)]Y_t + (1-\delta)K_t
\end{aligned}$$

From here, the steps follow exactly as they did in the answer to part (b) except we have a different constant in front of Y_t. The end result is that we get:

$$\begin{aligned}
k_{ss} &= \left(\frac{s + \tau(1-s)}{\delta + \tilde{x} + n + n\tilde{x}} \right)^{\frac{1}{1-\alpha}} \\
\frac{Y_{t,bg}}{L_{t,bg}} &= \tilde{A}_{t,bg} \left(\frac{s + \tau(1-s)}{\delta + \tilde{x} + n + n\tilde{x}} \right)^{\frac{\alpha}{1-\alpha}}
\end{aligned}$$

d. Private consumption will be given by $C_t = (1-s)Y_t^D = (1-s)(1-\tau)Y_t$. Plugging in our result from part (c) this gives us:

$$\begin{aligned}
\frac{C_{t,bg}}{L_{t,bg}} &= (1-s)(1-\tau)\tilde{A}_{t,bg} \left(\frac{s + \tau(1-s)}{\delta + \tilde{x} + n + n\tilde{x}} \right)^{\frac{\alpha}{1-\alpha}} \\
&= \frac{(1-s)\tilde{A}_{t,bg}}{(\delta + \tilde{x} + n + n\tilde{x})^{\frac{\alpha}{1-\alpha}}} (1-\tau)[s + \tau(1-s)]^{\frac{\alpha}{1-\alpha}}
\end{aligned}$$

The question of the optimal choice of τ is then just asking what τ maximizes the value of the above expression. Since everything before $(1-\tau)$ is a constant with respect to τ we ignore it in the calculation below. We proceed by taking the derivative with respect to τ and setting it equal to zero.

$$
\begin{aligned}
0 &= \frac{d}{d\tau}(1-\tau)[s+\tau(1-s)]^{\frac{\alpha}{1-\alpha}} \\
0 &= -[s+\tau(1-s)]^{\frac{\alpha}{1-\alpha}} + (1-\tau)\frac{\alpha}{1-\alpha}[s+\tau(1-s)]^{\frac{\alpha}{1-\alpha}-1}(1-s) \\
0 &= -1 + (1-\tau)\frac{\alpha}{1-\alpha}[s+\tau(1-s)]^{-1}(1-s) \\
1 &= \frac{\alpha}{1-\alpha}(1-\tau)[s+\tau(1-s)]^{-1}(1-s) \\
[s+\tau(1-s)] &= \frac{\alpha}{1-\alpha}(1-\tau)(1-s) \\
\tau(1-s) &= \frac{\alpha}{1-\alpha}(1-s) - \frac{\alpha}{1-\alpha}(1-s)\tau - s \\
\left(1+\frac{\alpha}{1-\alpha}\right)(1-s) &= \frac{\alpha}{1-\alpha}(1-s) - s \\
(1-\alpha+\alpha)(1-s)\tau &= \alpha(1-s) - s(1-\alpha) \\
(1-s)\tau &= \alpha - s\alpha - s + s\alpha \\
(1-s)\tau &= \alpha - s \\
\tau &= \frac{\alpha - s}{1-s}
\end{aligned}
$$

Let's consider if this expression makes sense. We know from class that when $s = \alpha$ consumption is maximized. In this case, when $s = \alpha$ then $\tau = 0$, essentially saying that if the private sector is already behaving optimally there is no room for government to improve. On the other hand, if $s = 0$ then all savings in the economy is done by the government and $\tau = \alpha$. Other than these extreme cases, though, is using τ better or worse than controlling s? Using our result for τ we have:

$$
\begin{aligned}
(1-s)(1-\tau)[s+\tau(1-s)]^{\frac{\alpha}{1-\alpha}} &= (1-s)\left(1-\frac{\alpha-s}{1-s}\right)\left[s+\frac{\alpha-s}{1-s}(1-s)\right]^{\frac{\alpha}{1-\alpha}} \\
&= (1-s-\alpha+s)[s+\alpha-s]^{\frac{\alpha}{1-\alpha}} \\
&= (1-\alpha)[\alpha]^{\frac{\alpha}{1-\alpha}}
\end{aligned}
$$

This is the same result we had in class when we had only total savings $s = \alpha$ and no government sector.