Spring 2016 Econ 164 Mid-term Exam Answer Key

Prof. Saki Bigio

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1. Consider the Solow model with population growth but no technology progress. Suppose at time 0 the TFP ('A' in the model) is suddenly increased permanently due to an economic reform. The following figure shows the dynamic path of TFP. Answer the following questions. (30 points)

a. What is the long run effect on capital per capita of the increase in TFP? Show graphically how you reach the conclusion. (15 points)

b. What is short run effect on capital per capita? Draw the dynamic path of capital per capita after the increase in TFP. (15 points)

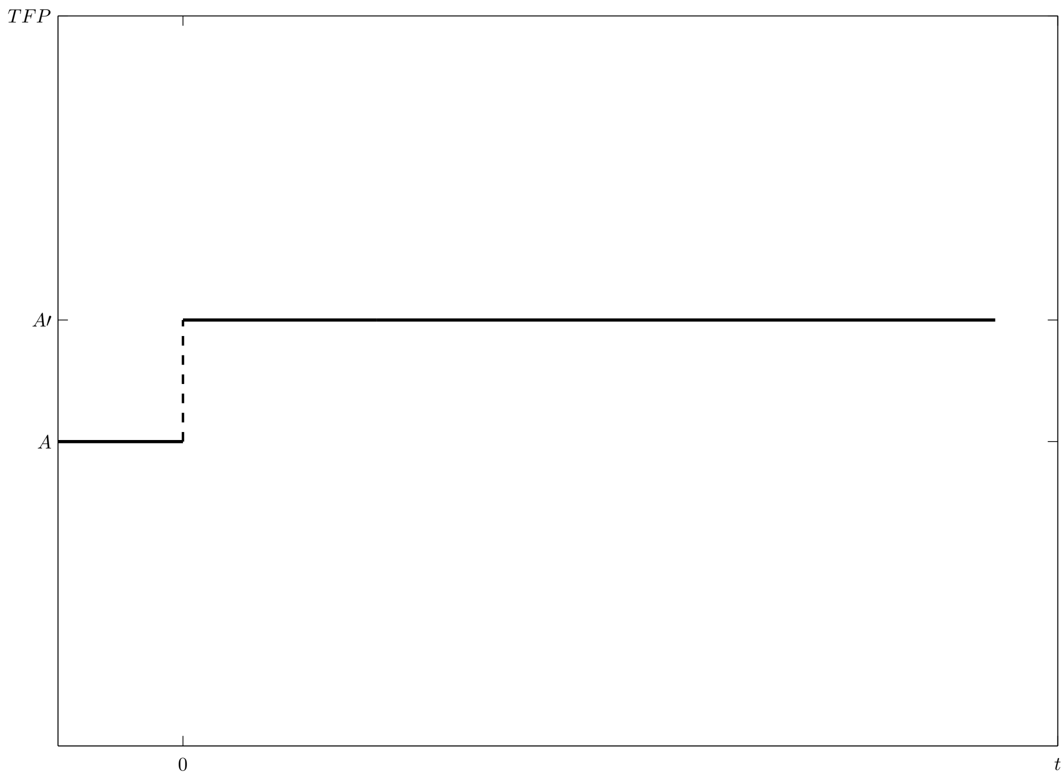

Figure 1: the dynamic path of TFP

Answer:

a. An increase in TFP raises gross saving at any k. The long run effect is summarized by a new steady state which is higher than the old one. The graph is shown below in Figure 2.

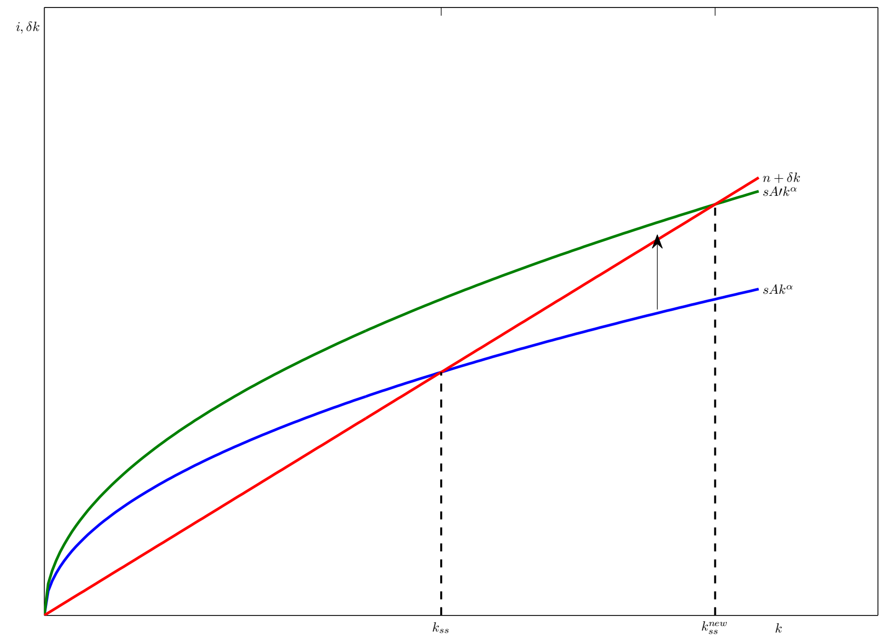

Figure 2: change in steady state

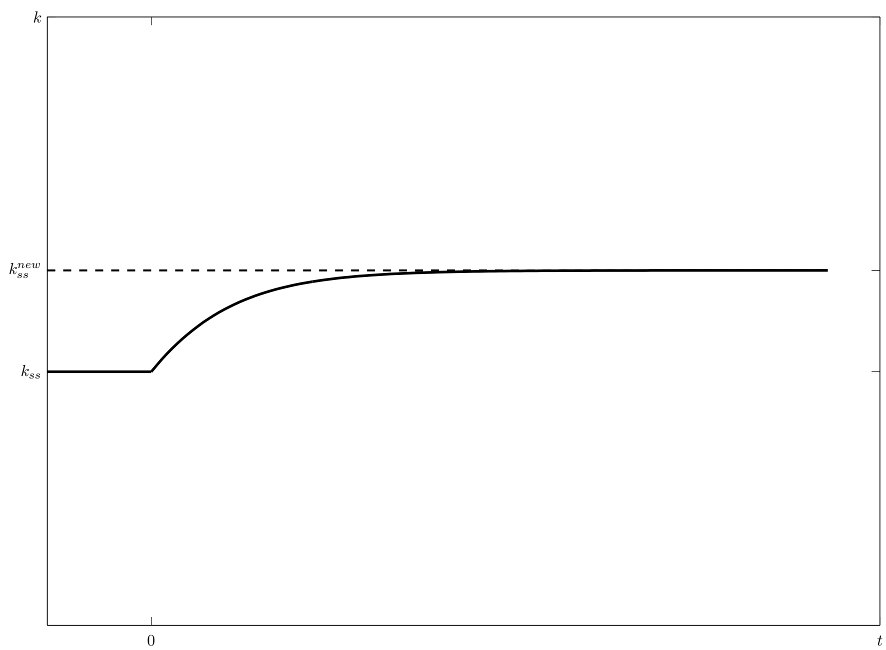

Figure 3: the dynamic path of k

b. In the short run, the economy experience an gradual increase in k. The speed of the increases slows down as the economy approaches the new steady state. The dynamic path is shown above in Figure 3.

2. We have learned that capital per capita in the Solow model is given by the following equation:

$$k_t = \left[\frac{\mathbf{sA}}{\boldsymbol{\delta}} + \left(k_0^{1-\alpha} - \frac{\mathbf{sA}}{\boldsymbol{\delta}}\right)\exp\left(-\left(1-\alpha\right)\boldsymbol{\delta}t\right)\right]^{\frac{1}{1-\alpha}}.$$

Answer the following questions using this equation. (25 points)

a. From this equation, find the steady state k_{ss}. (5 points)

b. Suppose we start with $k_0 = 0.7k_{ss}$, find an expression for the time required for the economy to be 2% away from the steady state. (10 points)

c. Let $\alpha = 0.33$ and $\delta = 0.08$, find an exact value for the time needed. (10 points)

Answer:

a. The steady state is reached as t approaches infinite, $k_{ss} = \left(\frac{sA}{\delta}\right)^{\frac{1}{1-\alpha}}$.

b. The time required is given as follows.

$$t = -\frac{\log\left(\frac{0.98^{1-\alpha}-1}{0.7^{1-\alpha}-1}\right)}{(1-\alpha)\delta}$$

c. Plugging in the numbers, the exact time needed is $t = 51.5$.

3. Let's consider the Solow model with population and technology growth. The steady state for capital per effective labor $\hat{k}_{ss}$ is given by

$$\hat{k}_{ss} = \left(\frac{s}{\delta + n + \tilde{x} + n\tilde{x}}\right)^{\frac{1}{1-\alpha}},$$

Answer the following questions. (20 points)

a. How will the steady state value change in response to a doubling of saving rate? Let the old and new steady state be k_{ss}^o and k_{ss}^n respectively, find a relationship between them. (5 points)

b. Let's assume $\alpha = 0.3, s = 0.15, \delta = 0.05, n = 0.01,$ and $\tilde{x} = 0.01$, find an exact value for the steady state. (5 points)

c. If the depreciation is increased to 0.08, what's the new steady state? Is it higher or lower than the original steady state? What's the intuition behind that? (10 points)

Answer:

a. $\frac{k_{ss}^n}{k_{ss}^o} = 2^{\frac{1}{1-\alpha}}$.

b. The steady state value is 2.96.

c. Under the new depreciation rate, the steady state value is 1.78. It is lower than the original value. It is because an increase in depreciation requires a larger return on capital to balance between gross saving and capital depreciation. The steady state is lowered to achieve a larger return.

4. The law of motion for aggregate capital in the Solow model is given by

$$K_{t+1} = sAK_t^{\alpha}L_t^{1-\alpha} + (1-\delta)K_t.$$

Population grows according to $L_{t+1} = (1+n)L_t$ and there is no technology growth. Answer the following questions. (25 points)

a. Rewrite the law of motion in terms of capital per capita k_t. Find the steady state for k_t. (10 points)

b. Find an expression for steady state per capita consumption. Find the golden rule saving rate(remember that the golden rule saving rate maximizes consumption per capita in steady state)? (10 points)

c. If the actual saving rate is above the golden rule, do you support a policy that reduces saving rate? (5 points)

a. In per capita terms, the law of motion is given by

$$k_{t+1} - k_t = \frac{sAk_t^\alpha - (n+\delta)k_t}{1+n}.$$

The steady state is $k_{ss} = \left(\frac{sA}{n+\delta}\right)^{\frac{1}{1-\alpha}}$.

b. The steady state consumption per capita is given by

$$c_{ss} = (1-s)Ak_{ss}^\alpha = (1-s)A\left(\frac{sA}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

It is maximized when $s = \alpha$.

c. Yes. It's a good policy as it raises consumption for all generations.