Spring 2017 Econ 164 Mid-term Exam

Prof. Saki Bigio

1. Consider the Solow model with no population growth but with technology progress. Suppose at time 0 the the number of workers in the economy drop because a portion of workers decide to retire from the work force. This is what happened in the US since 2008 when the "baby-boomers" started to retire. The following figure shows the labor force participation rate since 1990. Answer the following questions. (45 points)

a. What is the long-run effect on the level of output per worker and its growth rate? (15 points)

b. What is short run effect on total output? Draw a picture that plots the expected behavior of log output against time on the x-axis. (15 points)

c. Over the last few years, there's been a concern about the decline in productivity growth in the US over the last decade. Explain how you could use the Solow model to decompose how much of the deviation of GDP from trend has to do with "baby-boomers" retiring in mass in 2008. (15 points)

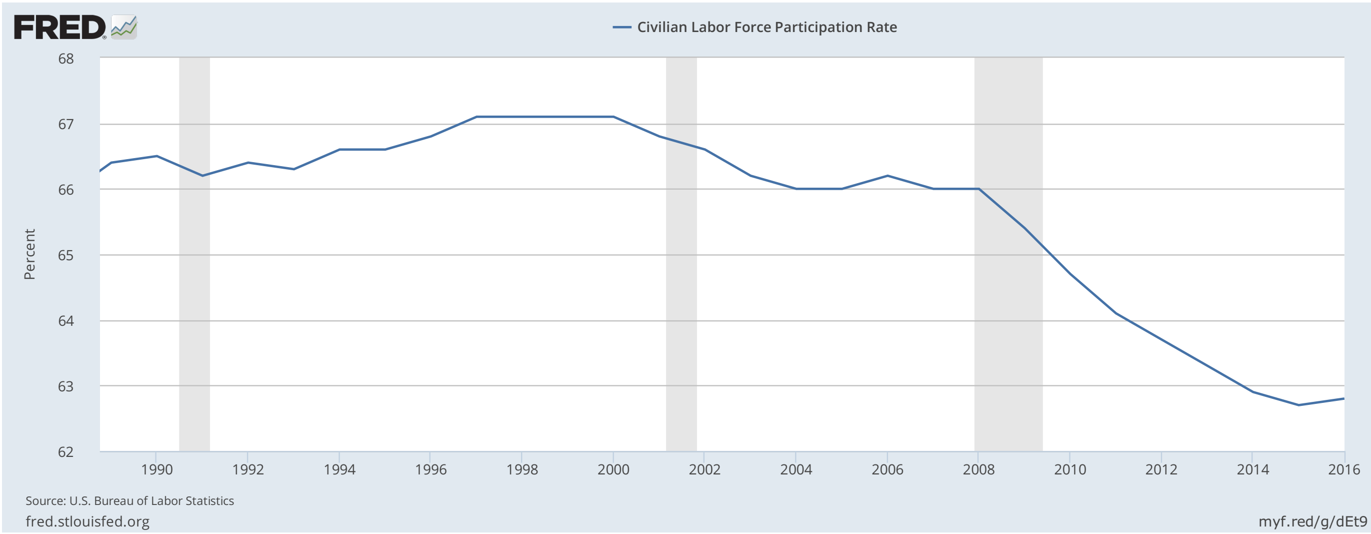

Figure: US Labor Participation Rate: 1990-present

Answers:

a. There would be no effect. In the long run we would reach the new balanced growth path, where output per worker grows at the same rate of technological growth.

b. Output will drop in the initial period because of the reduction in the workforce. In the same period the capital per efficiency unit of work has to go above its steady state value. In order to go back to the blanced growth path, the economy will divest some capital (i.e. reduce the amount of total capital) in the short term. Therefore, in the periods following the decline in the workforce, total output will grow at a smaller rate than the rate of technological growth. Over time, output growth will approach the growth rate of technology.

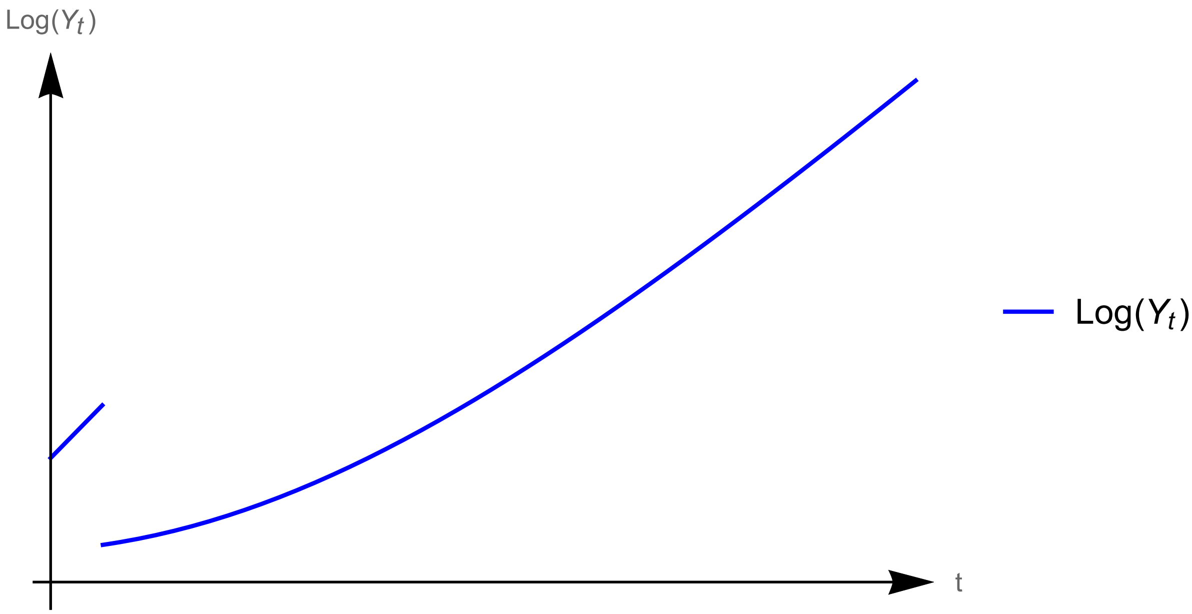

Figure 2: Log Output

c. According to the Solow decomposition Δ. Using the model with no population growth and technological process, we would conclude that labor had no role in growth (since $\Delta\%L = 0$) and that $\Delta\%A$ and $\Delta\%K$ grow the at constant rate of technology $\tilde{x}$. So the trend growth would be $\tilde{x}$. Following a reduction in the labor force ($\Delta\%L < 0$), we would obtain growth rates below the trend. In particular, the growth rate would decline by an amount equal to the percentage decrease in the labor force multiplied by the labor share $1 - \alpha$.

2. We have learned that capital per efficiency units of labor in the Solow model is given by the following equation:

$$\tilde{k}_t = \left[\frac{\mathrm{s}}{\delta + g} + \left(k_0^{1-\alpha} - \frac{\mathrm{s}}{\delta + g} \right) \exp\left(-\left(1-\alpha\right)\left(\delta+g\right)t \right) \right]^{\frac{1}{1-\alpha}}.$$

Answer the following questions using this equation. Use the following standard numbers for the Solow Model when necessary $s = 0.15$, $\alpha = 0.33, g = 0.025$ and $\delta = 0.08$. (25 points)

a. From this equation, find the steady state $\tilde{k}_{ss}$ for capital per efficiency units of labor. (5 points)

b. Suppose Japan started with $k_0 = 0.5k_{ss}$ after World-War 2. Assume that the US was already in steady-state by then and remained on a balanced growth path. Find an expression for the time required for the Japan to reach 1% away from US GDP per capita. (10 points)

c. Find an exact value for the time that Japan needed to catch-up with the US. Do you think the Solow model does a good job explaining the patterns in the data? (10 points)

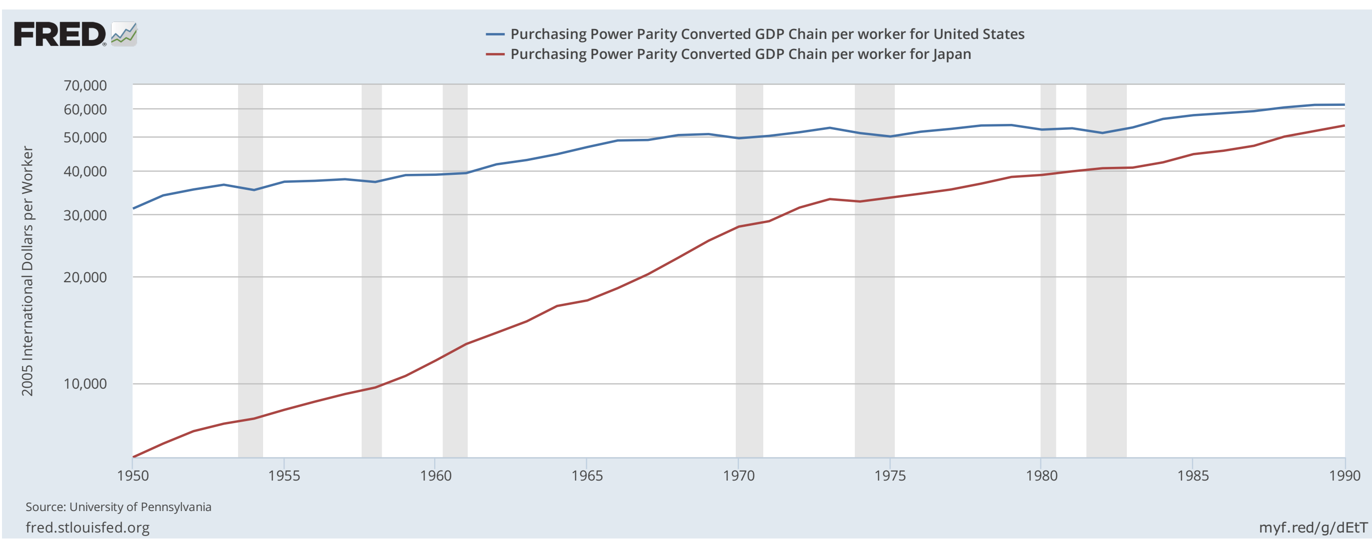

Figure: GDP per Worker of US and Japan: 1950-1990

Answers:

a.

$$\tilde{k}_{ss} = \left(\frac{s}{\delta + g}\right)^{\frac{1}{1-\alpha}}$$

b. We want $\tilde{k}_t = 0.99\tilde{k}_{ss}$, therefore

$$0.99^{1-\alpha}\left(\frac{s}{\delta + g}\right) = \frac{\mathbf{s}}{\delta + g} + \left(0.5^{1-\alpha}\left(\frac{s}{\delta + g}\right) - \frac{\mathbf{s}}{\delta + g}\right)\exp\left(-\left(1-\alpha\right)\left(\delta + g\right)t\right)$$

Which simplifies to

$$0.99^{1-\alpha} - 1 = \left(0.5^{1-\alpha} - 1\right)\exp\left(-\left(1-\alpha\right)\left(\delta + g\right)t\right)$$

Divide and take the log

$$\log\frac{0.99^{1-\alpha} - 1}{0.5^{1-\alpha} - 1} = -\left(1-\alpha\right)\left(\delta + g\right)t$$

Thus

$$t = -\frac{1}{\left(1-\alpha\right)\left(\delta + g\right)}\log\frac{0.99^{1-\alpha} - 1}{0.5^{1-\alpha} - 1}$$

c. Replace the numbers in the previous formula to get $t \simeq 57.05$

3. Consider the Solow model a different savings technology. Assume that the labor supply L_t is constant and equal to L. Let savings be given by:

$$S_t = \max\{sY_t - \bar{c}L_t, 0\}$$

where s is a constant savings rate and c̄ is a subsistence consumption level.

Also, let output be given by:

$$Y_t = A_t\left(F\left(K_t, L\right) + \bar{e}L\right)$$

where $\bar{e}$ is a small endowment of natural resources. The production function is $F\left(K_t, L_t\right) = K_t^{\alpha} L^{1-\alpha}$.

Let the capital accumulation equation be given by:

$$K_{t+1} = S_t + (1-\delta)\,K_t$$

For the time being, A is also constant. Answer the following questions. (30 points)

a. Draw total savings S_t as a function of Y_t for some value $\bar{c} > 0$ and some other value $\bar{c} = 0$. (5 points)

b. Draw aggregate capital stock K_{t+1} as a function of K_t for some values $\{\bar{c}, \bar{e}\} > 0$ and some other value $\{\bar{c}, \bar{e}\} = 0$. (5 points)

c. Show that if $sA\bar{e} < \bar{c}$, there exists a steady state level of capital $K_{ss} = 0$. (5 points)

d. Argue with a picture how if the economy satisfies $sA\bar{e} < \bar{c}$, if the economy begins with a low capital stock per worker, it necessarily converges to $K_{ss} = 0$ and consumption per capita equals $c_{ss} = A\bar{e}$. (5 points)

e. Explain how if $K_0 = 0$, if there is a TFP jump such that now $sA\bar{e} > \bar{c}$, the economy can break apart from the subsistence condition in the previous question. (10 points)

Answers:

a.

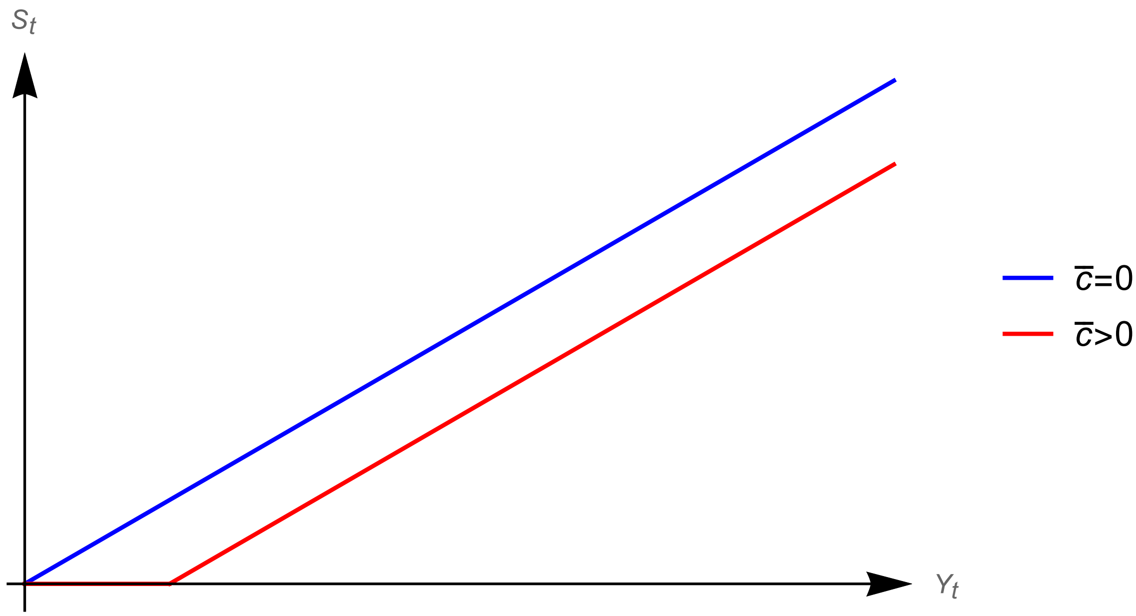

Figure 4: Savings

b.

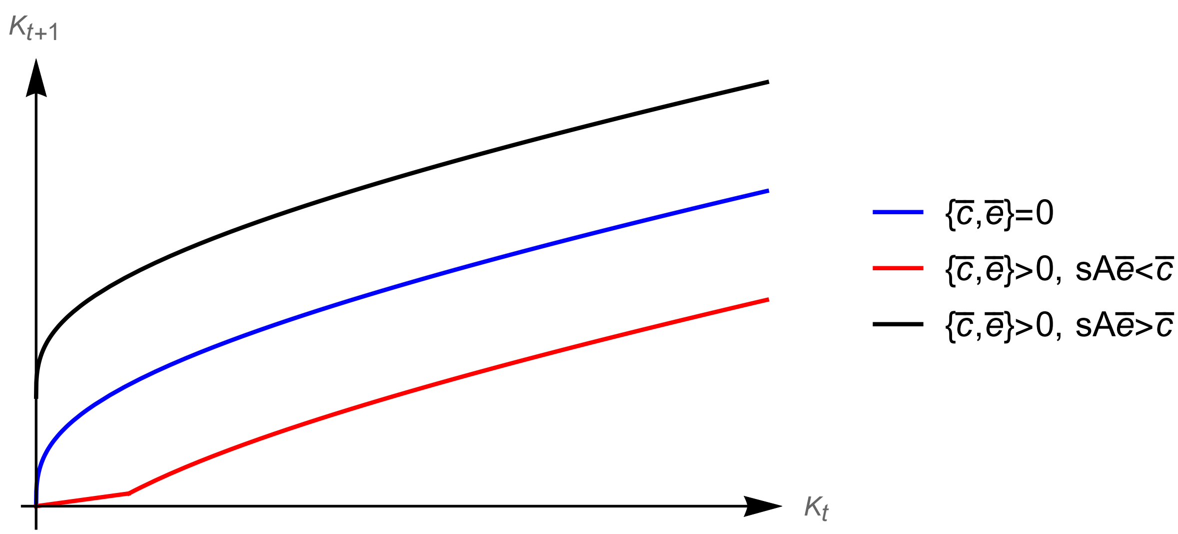

Figure 5: Capital Accumulation

c. When $sA\bar{e} < \bar{c}$, there are values of K_t such that $sA_t\left(K^{\alpha}L^{1-\alpha} - \bar{e}L\right) - \bar{c}L < 0$.

That is, for those values $S_t = 0$. Clearly $K_t = 0$ satisfies this condition, so the law of motion for capital becomes $K_{t+1} = (1 - \delta)K_t$. Replacing $K_t = 0$ returns $K_{t+1} = 0$, which is the definition of steady state.

d. See the picture in part b. When the level of capital is low enough (i.e. $S_t = 0$) the capital accumulation becomes $K_{t+1} = (1 - \delta)K_t$. Therefore, capital slowly depreciates and converges to zero. At $K_{ss} = 0$ we have $Y_{ss} = Ae\bar{L}$ and $S_t = 0$, so it must be the case that all production is consumed, or $C_{ss} = Ae\bar{L}$. Then consumption per capita is $c_{ss} = {}^{C_{ss}}/_{\bar{L}} = A\bar{e}$.

e. See the picture in part b. Say you start from $K_0 = 0$. Then capital would immediately jump to $K_1 = sAe\bar{L} - c\bar{L} > 0$. Capital accumulation would then follow the same path of the standard Solow model until it reaches a steady state with $K_{SS} > 0$.