Spring 2019 Econ 164 Mid-term Exam

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May 2, 2019

The exam has 4 questions for 150 points in total. You have 75 minutes to finish the exam.

NOTE THAT POINTS VARY BETWEEN QUESTIONS.

DO NOT OPEN UNTIL THE EXAM BEGINS

Name:

UID:

Grade:

1. Steady State Comparisons (30 points). Consider the Solow model with population and technology growth. The steady state for capital per effective labor $\tilde{k}_{ss}$ is given by

$$\tilde{k}_{ss} = \left(\frac{s}{\delta + n + \tilde{x} + n\tilde{x}}\right)^{\frac{1}{1-\alpha}},$$

Let's assume $\alpha = 0.3$, $s = 0.2$, $\delta = 0.05$, $n = 0.01$, and $\tilde{x} = 0.01$. Answer the following questions. (20 points)

a. Find an exact value for capital in the steady state. (7.5 points)

b. How will the steady state value $\hat{k}_{ss}$ change in response to a 20% decrease in the saving rate? Let the old and new steady state values be $\tilde{k}^o_{ss}$ and $\tilde{k}^n_{ss}$ respectively, and describe the relationship between them. (7.5 points)

c. If the coefficient of decreasing returns, α is increased to 0.4, what is capital per effective labor in the new steady state? Is it higher or lower than in the original steady state? Provide the intuition for your result using any of the graphical tools learned in class. (15 points)

a.

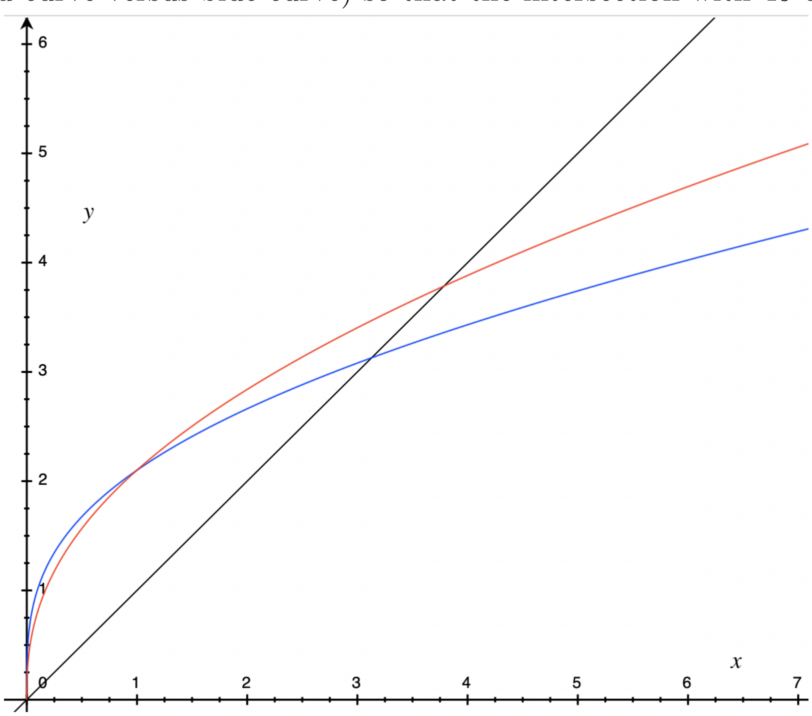
$$k^o_{ss} = \left(\frac{0.2}{0.05 + 0.01 + 0.01 + 0.0001}\right)^{\frac{1}{0.7}} \approx 4.47142$$

b. Elasticity of capital per effective worker with respect to saving rate is $1/(1-\alpha) \approx 1.429$. So 20% decrease of saving rate results in $20 * 1.429 \approx 28.56\%$ decrease in capital per effective worker. More precisely, $k^o_{ss} = \left(\frac{s}{\delta+n+\tilde{x}+n\tilde{x}}\right)^{\frac{1}{1-\alpha}}$ while $k^n_s = \left(\frac{0.8s}{\delta+n+\tilde{x}+n\tilde{x}}\right)^{\frac{1}{1-\alpha}}$ giving us that $\frac{k^n_{ss}}{k^o_{ss}} = (0.8)^{\frac{1}{1-\alpha}}$.

c.

$$k^n_{ss} = \left(\frac{0.2}{0.05 + 0.01 + 0.01 + 0.0001}\right)^{\frac{1}{0.6}} \approx 5.73922$$

Capital per effective worker is now higher in steady state. Intuitively, this is because diminishing returns to capital kick in later than with lower α, so the capital accumulation line is less curved (red curve versus blue curve) so that the intersection with 45 degree line happens earlier:



Note however that the effect would have been opposite if $\hat{k} < 1$ because for low values of $\hat{k}$ the curve being less curved results in downward shift. But this is not the relevant case for reasonable values of parameters, so it is ok if you missed this case and drew a curve scaled upwards.

2. Growth Accounting (40 points).

In this exercise, we will do growth accounting for the United States. Below is the table with data on real GDP, capital stock, employment, and compensation share of labor for years 1960, 1980, 2000, from the Penn World Tables:

Year	Real GDP, \$bn.	Capital Stock, \$bn.	Employment, mil.	Labor share
1960	3,198,295	12,368,243	70.98	0.6367
1980	6,631,990	24,366,128	103.20	0.6243
2000	12,883,895	41,503,672	138.77	0.6370

a. Compute 6 numbers: percentage change in output, capital, and labor force in 1960-1980 and in 1980-2000, respectively. What happened to output growth over time? (10 points)

b. For growth accounting, we will need an estimate of parameter α which we will assume to be constant over time. How can we justify this assumption? Give your own estimate of α based on the data provided. (7.5 points)

c. Using your answers to a and b, use the Solow decomposition from class to compute percentage change in technology level A_t in 1960-1980 and in 1980-2000. Then, compute contributions of capital accumulation and technology to output growth in 1960-1980 and in 1980-2000. (7.5 points)

d. In which time period does technology have higher contribution to growth? Comment on what was hapenning to US economy over time using the Solow model with technology growth. (7.5 points)

e. Provide one criticism of the approach above. (Hint: Is there any estimation bias of this method? Under what circumstances can this method be problematic to answer the above questions?) (7.5 points)

a. We compute percentage change using the well-known formula for growth rate:

$$\%\Delta Y_{1960\to 1980} = \frac{Y_{1980} - Y_{1960}}{Y_{1960}} * 100\%$$

The resulting numbers approximately are:

Period	$\%\Delta Y$	$\%\Delta K$	$\%\Delta L$
1960-1980	107.36	97.00	45.39
1980-2000	94.27	70.33	34.47

Output growth slowed down over time.

b. We assume that it is constant because in the model it corresponds to the share of national income paid to capital owners (or 1 minus labor share), and as the last column of the table shows, the share of income paid to labor (and hence to capital) is indeed remarkably constant in the data. We can estimate it as the simple average of three numbers given:

$$\alpha = 1 - (0.6367 + 0.6243 + 0.6370)/3 \approx 0.363$$

c. Solow decomposition is obtained from Cobb-Douglas production function:

$$\%\Delta A = \%\Delta Y - \alpha * \%\Delta K - (1 - \alpha) * \%\Delta L$$

In 1960-1980, we obtain

$$\%\Delta A = 107.36 - 0.363 * 97.00 - 0.637 * 45.39 = 43.23\%$$

In 1980-2000,

$$\%\Delta A = 94.27 - 0.363 * 70.33 - 0.637 * 34.47 = 46.78\%$$

Contribution of capital and technology in 1960-1980;

$$\frac{\alpha * \%\Delta K}{\%\Delta Y} = 32.80\% \qquad \frac{\%\Delta A}{\%\Delta Y} = 40.27\%$$

Contribution of capital and technology in 1980-2000;

$$\frac{\alpha * \%\Delta K}{\%\Delta Y} = 27.08\% \qquad \frac{\%\Delta A}{\%\Delta Y} = 49.62\%$$

d. Technology has higher contribution to growth in later period, whereas contribution of capital is larger in earlier period. This makes sense from the viewpoint of Solow model: at the low level of development capital accumulation matters a lot, but in the long run it is the technology that matters for output growth.

e. There are many criticisms of this type of decomposition. Most of them are concerned with improper measurement of capital or labor. For example, when we measure employment by the total number of workers we ignore the possibility that number of weekly hours worked changes. So when workers start working more every week, we will see higher output without change of total labor force and we will erroneously attribute it to increase in technology A_t. We also lump all occupations of workers in one single number. Output may go up because larger proportion of workers become high-skilled but the total number of workers does not change. There are similar issues with capital: what is measured by statistical agency is the value of physical capital like structures and equipment owned by firms. There can be other types of intangible capital that cannot be measured easily (like expertise of firms in the industry or well-established customer network). On top of that, firms may choose different utilization rate of capital: they might have the same number of machines as before, but use only a fraction of them to produce lower output than usually. We will see decrease in output without change of value of capital and attribute it to fall in technology A_t whereas it is simply decrease of utilization rate by firms.

3. Convergence (40 points).. We have learned that capital per efficiency units of labor in the Solow model with no population growth and with technology growing at rate g is given by the following equation:

$$\tilde{k}_t = \left[\frac{\mathrm{s}}{\delta+g} + \left(\tilde{k}_0^{1-\alpha} - \frac{\mathrm{s}}{\delta+g}\right)\exp\left(-\left(1-\alpha\right)\left(\delta+g\right)t\right)\right]^{\frac{1}{1-\alpha}}.$$

Answer the following questions using this equation. Use the following standard numbers for the Solow Model when necessary: $s = 0.15$, $\alpha = 0.33$, g=0.025 and $\delta = 0.08$.

a. From this equation, find the steady state $\tilde{k}_{ss}$ for capital per efficiency units of labor. (7.5 points)

For questions (b) and (c), suppose that a country has just finished a war at time $t = 0$. During the war, the government had imposed strict rationing and used the extra resources to build up industrial capacity to help the war effort. Once the war is over, the government allows savings to return to its normal level but the capital stock is high, so that $\tilde{k}_0 = 2\tilde{k}_{ss}$.

b. Find an expression (in terms of the parameters of the model) for the time it will take for $\tilde{k}_t$ to be 1% **above** its steady state. (10 points)

c. Find an exact numeric value for the time needed for $\tilde{k}_t$ to reach 1% above its steady state. (7.5 points)

d. This model exhibits convergence of capital per effective worker. Is it unconditional convergence or conditional convergence? Explain your answer. (7.5 points)

e. Suppose there are two countries that both grow according to the Solow model above and have same initial aggregate capital stock and population size, except that country 1 has a higher initial technology level A_0 than country 2. How does the growth rate of capital in each country change over time? Which country grows faster? Plot the time paths of two countries' growth rates of capital per worker (not efficiency worker) in a figure and explain your answer by words. (7.5 points).

Answer:

a. From last equation,

$$\lim_{t\to\infty}\tilde{k}_t^{1-\alpha} = \frac{\mathrm{s}}{\delta+g} + \lim_{t\to\infty}\left(\tilde{k}_0^{1-\alpha} - \frac{\mathrm{s}}{\delta+g}\right)\exp\left(-\left(1-\alpha\right)\left(\delta+g\right)t\right) \tag{1}$$

Assuming $\tilde{k}_0 > 0$,

$$\lim_{t\to\infty}\left(\tilde{k}_0^{1-\alpha} - \frac{\mathrm{s}}{\delta+g}\right)\exp\left(-\left(1-\alpha\right)\left(\delta+g\right)t\right) = 0$$

Hence

$$\lim_{t\to\infty}\tilde{k}_t = \left(\frac{\mathrm{s}}{\delta+g}\right)^{\frac{1}{1-\alpha}}$$

b. We want $\tilde{k}_t = 1.01\tilde{k}_{ss}$, therefore

$$1.01^{1-\alpha}\left(\frac{s}{\delta+g}\right) = \frac{\mathrm{s}}{\delta+g} + \left(2^{1-\alpha}\left(\frac{s}{\delta+g}\right) - \frac{\mathrm{s}}{\delta+g}\right)\exp\left(-\left(1-\alpha\right)\left(\delta+g\right)t\right)$$

Which simplifies to

$$1.01^{1-\alpha} - 1 = \left(2^{1-\alpha} - 1\right)\exp\left(-\left(1-\alpha\right)\left(\delta+g\right)t\right)$$

Divide and take the log

$$\log\left(\frac{1.01^{1-\alpha} - 1}{2^{1-\alpha} - 1}\right) = -\left(1-\alpha\right)\left(\delta+g\right)t$$

Thus

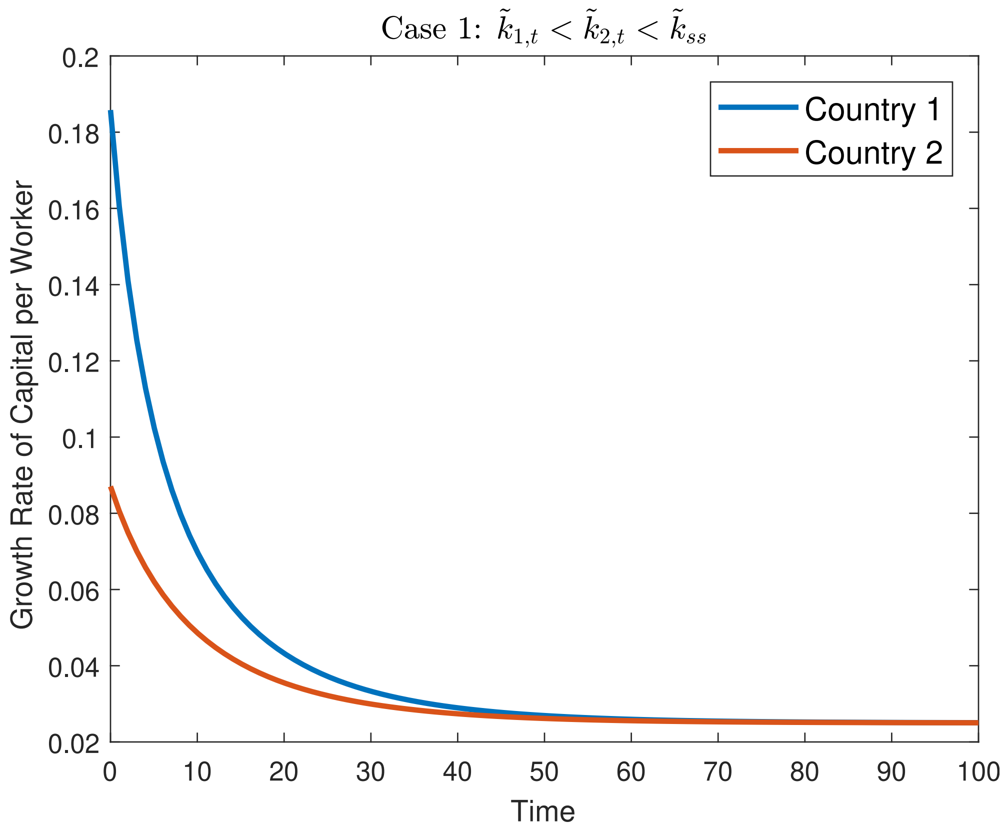
$$t = -\frac{1}{(1-\alpha)(\delta+g)} \log\left(\frac{1.01^{1-\alpha}-1}{2^{1-\alpha}-1}\right)$$

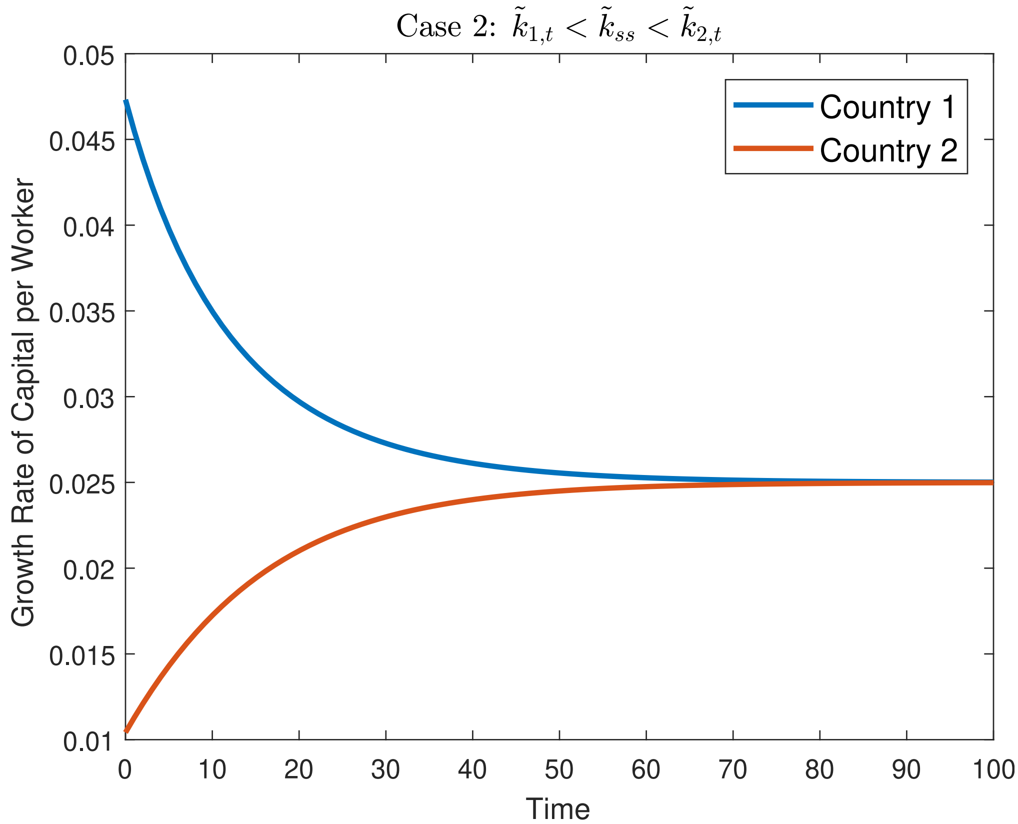
c. Replace the numbers in the previous formula to get $t \simeq 61.95$.

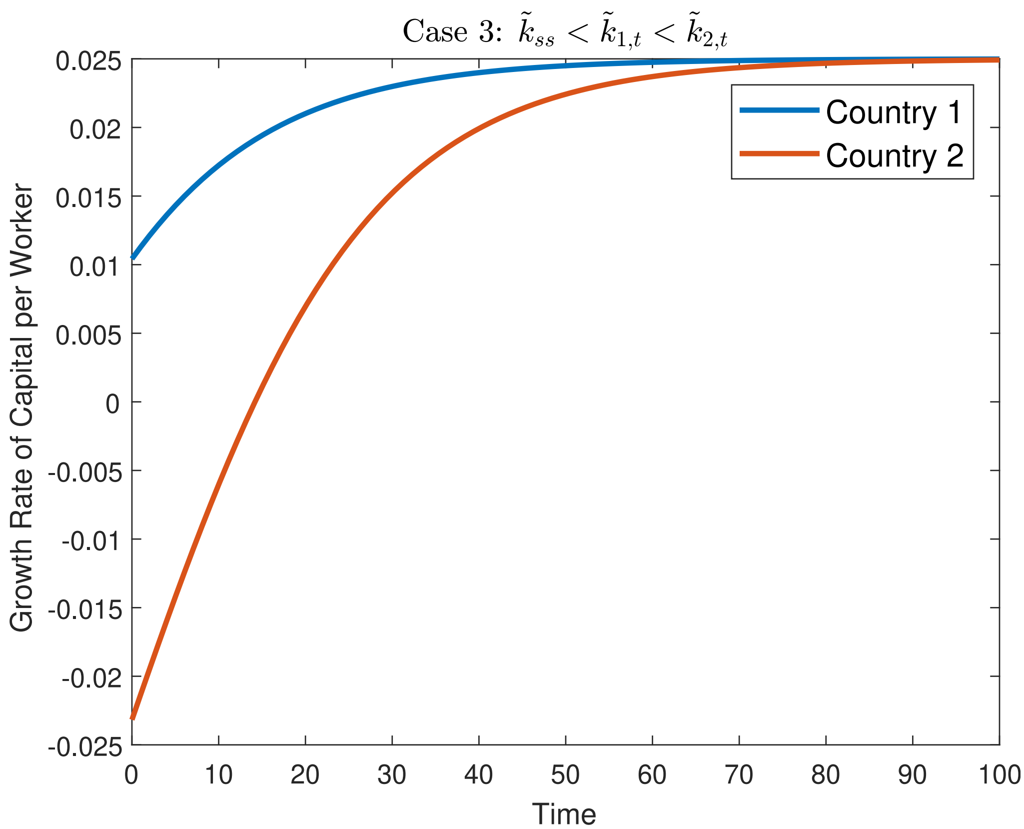
d. This model exhibits conditional convergence. Given same values of model parameters s, α, g and δ, the capital per effective worker converges to the same steady state for any initial value. However, it will converge to different steady states if the parameters change.

e. In this model, the growth rate of capital and the growth rate of capital per worker are the same because there is no population growth. Both of them are equal to the growth rate of capital per efficiency worker plus technology growth rate g. If a country's initial capital per efficiency worker is higher than its steady state value, then its growth rate of capital is below g and gradually increases and converges to g. This is because its capital per efficiency worker gradually decreases and converges to steady state value. If a country's initial capital per efficiency worker is lower than steady state, then its growth rate of capital is above g and gradually decreases and converges to g. This is because its capital per efficiency worker gradually increases and converges to steady state value.

For the two countries, country 1 has a lower initial capital per efficiency labor. If everything else is equal, a country with lower initial capital per efficiency labor has a higher marginal product of capital, thus the output and investment grow at a faster speed, and so does capital stock. Therefore, country 1 grows faster. The time paths of two countries' growth rates of capital per worker are plotted in the following figures (we plot three possible cases):




Case 2: $\tilde{k}_{1,t} < \tilde{k}_{ss} < \tilde{k}_{2,t}$
Country 1
Country 2
Growth Rate of Capital per Worker
Time
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0
10
20
30
40
50
60
70
80
90
100


Case 3: $\tilde{k}_{ss} < \tilde{k}_{1,t} < \tilde{k}_{2,t}$
Country 1
Country 2
Growth Rate of Capital per Worker
Time
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
0
10
20
30
40
50
60
70
80
90
100

4. Myopic and Forward-looking Governments (40 points). Consider a version of the discrete-time Solow model with a Government, which can be either Myopic or Forward-looking. The law of motion for aggregate capital in the Solow model is given by

$$K_{t+1} = I_t + (1-\delta) K_t,$$

where I_t is total investment in this economy. Defind I_t^{prv} and I_t^G as the private investment and government investment, respectively. Of course,

$$I_t = I_t^{prv} + I_t^G.$$

We assume that private investment, I^{prv}, is a constant proportion of disposable income:

$$I_t^{prv} = sY_t^D$$

and Y_t^D is disposable income, given by:

$$Y_t^D = Y_t - T_t,$$

where T_t is the income tax payment, and we assume $T_t = \tau Y_t$, with $\tau \in [0,1)$ the tax rate. Moreover, we assume the production function is Cobb-Douglas, i.e. $Y_t = AK_t^\alpha L_t^{1-\alpha}$. There is no technology growth, but the population size L_t grows at constant rate n.

a. Provide an expression for private investment only as a function of total income. Then use this equation to write the law of motion for capital per worker. You can use i_t^G to denote the government investment per worker. (5 points)

b. Now suppose the economy is in a steady state where $\tau = I^G = 0$ before period $\hat{t}$. Starting period $\hat{t}$, the government would like to raise workers' consumption level by fiscal policy. Assume the government is myopic. That is, starting from period $\hat{t}$, the government decides to impose a small positive tax rate τ, and transfer all the tax income back to individuals in the form of consumption. You can think of this as the government provides public service to households and the public service adds to households' consumption. Provide an expression for the consumption per worker, and find out GDP per capita, consumption per capita, and capital stock per capita in the new steady state. (7.5 points)

c. Assume that we are initially below Golden rule, i.e. there is too little saving in the economy. Following the previous question, draw the transition path of consumption per capita starting from period $\hat{t}$. Does the policy increase consumption in short run? in long run? (10 points) (Hint: we can think about the model described above as the standard Solow model with population growth, but where the total saving rate is the function of tax rate and private saving rate. Therefore, your task is to figure out whether total saving rate of the economy increases or decreases in response to tax.)

d. Now we consider a forward-looking government. That is, starting from time $\hat{t}$, the government imposes a small positive tax rate τ, and invest all the tax income into capital. Find out GDP per capita, consumption per capita, and capital stock per capita in the new steady state. (7.5 points)

e. Assume again that we are initially below Golden rule, i.e. there is too little saving in the economy. Following the previous question, draw the transition path of consumption per capita starting from period $\hat{t}$. Does the policy increase consumption in short run? in long run? (10 points)

Answer:

a. The private investment is $I^{prv} = sY_t^D = s(Y_t - G_t) = s(Yt - \tau Y_t) = s(1-\tau)Y_t$. Total investment would just be this plus government investment, but we do not yet know what the government chooses for I_t^G. Thus the law of motion for capital per worker can be derived as follows.

$$\begin{aligned}
K_{t+1} &= I_t^{prv} + I_t^G + (1-\delta)K_t \\
K_{t+1} &= s(1-\tau)Y_t + I_t^G + (1-\delta)K_t \\
(1+n)\frac{K_{t+1}}{L_{t+1}} &= s(1-\tau)\frac{Y_t}{L_t} + \frac{I_t^G}{L_t} + (1-\delta)\frac{K_t}{L_t} \\
(1+n)k_{t+1} &= s(1-\tau)y_t + i_t^G + (1-\delta)k_t
\end{aligned}$$

b. The consumption per worker is $c_t = C_t/L_t = \left(Y_t - T_t - I_t^{prv} + T_t\right)/L_t = \left[1 - s(1-\tau)\right] y_t$. The new steady state capital per worker is obtained from the law of motion in part a by replacing i_t^G with 0:

$$(1+n)\,k_{ss} = s(1-\tau)y_{ss} + (1-\delta)k_{ss}$$

By assumption of Cobb-Douglas production function, the steady-state GDP per capita is $y_{ss} = k_{ss}^{\alpha}$ (so we implicitly assume $A = 1$), giving us

$$(1+n)\,k_{ss} = s(1-\tau)k_{ss}^{\alpha} + (1-\delta)k_{ss}$$

$$k_{ss} = \left(\frac{s(1-\tau)}{\delta+n}\right)^{\frac{1}{1-\alpha}}$$

The GDP per capita and consumption per capita in new steady state are $y_{ss} = \left(\frac{s(1-\tau)}{\delta+n}\right)^{\frac{\alpha}{1-\alpha}}$ and $c_{ss} = \left[1 - s(1-\tau)\right]\left(\frac{s(1-\tau)}{\delta+n}\right)^{\frac{\alpha}{1-\alpha}}$.

c. Before $\hat{t}$, the steady state consumption per capita is $c_{ss}^{before} = (1-s)\left(\frac{s}{\delta+n}\right)^{\frac{\alpha}{1-\alpha}}$, and $y_{ss}^{before} = \left(\frac{s}{\delta+n}\right)^{\frac{\alpha}{1-\alpha}}$. At the moment right after $\hat{t}$, consumption per capita is

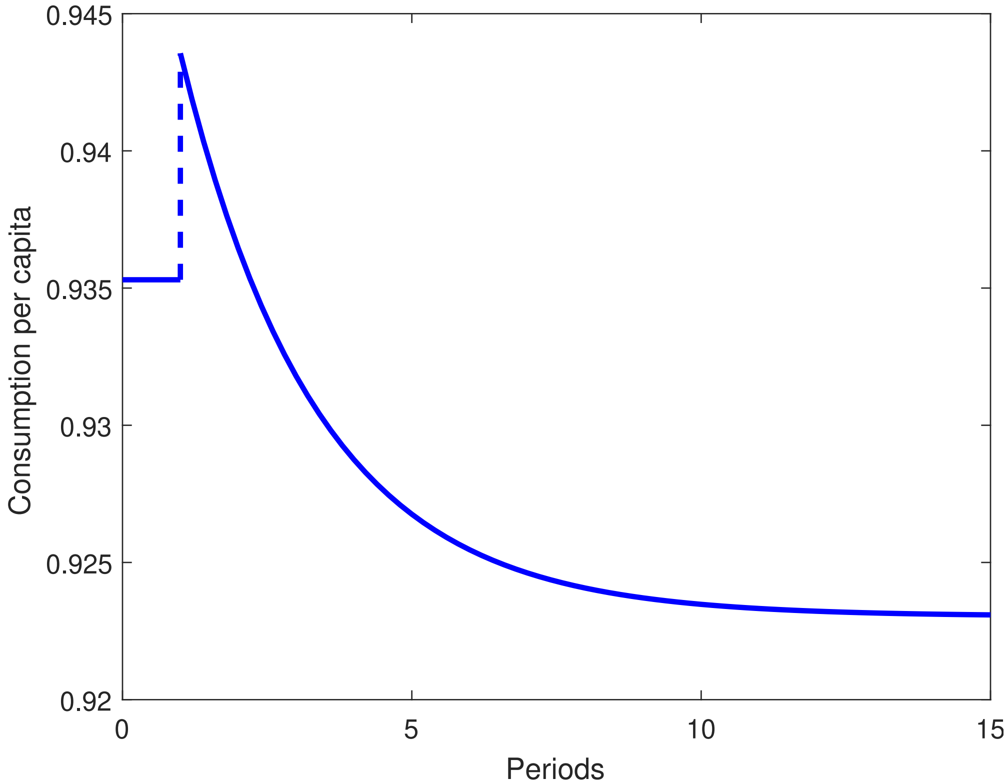
$$\left[1 - s(1-\tau)\right] y_{ss}^{before} = (1 - s + s\tau)\left(\frac{s}{\delta+n}\right)^{\frac{\alpha}{1-\alpha}} > c_{ss}^{before}.$$

Thus this policy can increase consumption in short run.

In long run, the new steady-state consumption per capita c_{ss} is given in part b. Taking first-order differentiation of $c_{ss}$with respect to τ yields

$$\begin{aligned}
\frac{\partial c_{ss}}{\partial \tau} &= s\left(\frac{s(1-\tau)}{\delta+n}\right)^{\frac{\alpha}{1-\alpha}} - \left[1 - s(1-\tau)\right]\frac{\alpha}{1-\alpha}\left(\frac{s(1-\tau)}{\delta+n}\right)^{\frac{\alpha}{1-\alpha}}\frac{1}{1-\tau} \\
&= -\left(\frac{s(1-\tau)}{\delta+n}\right)^{\frac{\alpha}{1-\alpha}}\left[\frac{\alpha - s\,(1-\tau)}{(1-\alpha)\,(1-\tau)}\right]
\end{aligned}$$

Since we assume s is below Golden rule, i.e. $s < \alpha$, and $\tau > 0$, the partial derivative $\frac{\partial c_{ss}}{\partial \tau} < 0$. This implies that for any $\tau > 0$, $c_{ss}|_{\tau} < c_{ss}|_{\tau=0} = c_{ss}^{before}$. Thus this policy decreases consumption in long run. The transition path is plotted in the figure below.



d. Now we have $i_t^G = \tau y_t$ instead of being zero, and the transfer to consumption is zero. This gives us:

$$c_{ss} = y_{ss} - \tau y_{ss} - i_{ss}^{prv} = (1-s)(1-\tau) y_{ss}$$
$$(1+n)k_{ss} = [s + \tau(1-s)]y_{ss} + (1-\delta)k_{ss}$$

From here, the steps follow exactly as they did in the answer to part b except we have a different constant in front of y_{ss}. The end result is that we get:

$$k_{ss} = \left(\frac{s + \tau(1-s)}{\delta + n}\right)^{\frac{1}{1-\alpha}}$$

Thus the GDP per capita is $y_{ss} = \left(\frac{s+\tau(1-s)}{\delta+n}\right)^{\frac{\alpha}{1-\alpha}}$ and consumption per capita is

$$c_{ss} = (1-s)(1-\tau)\left(\frac{s+\tau(1-s)}{\delta+n}\right)^{\frac{\alpha}{1-\alpha}}.$$

e. We follow the steps in part c. The steady state before $\hat{t}$ is the same as described in part c. At the moment right after $\hat{t}$, consumption per capita is

$$(1-s)(1-\tau)\, y_{ss}^{before} = (1-s)(1-\tau)\left(\frac{s}{\delta+n}\right)^{\frac{\alpha}{1-\alpha}} < c_{ss}^{before}$$

Thus this policy decreases consumption in short run.

In the long run, the new steady-state consumption per capita c_{ss} is given in part d. Take first-order derivative with respect to τ, we obtain

$$\begin{aligned}\frac{\partial c_{ss}}{\partial \tau} &= -(1-s)\left(\frac{s+\tau(1-s)}{\delta+n}\right)^{\frac{\alpha}{1-\alpha}} + \frac{\alpha(1-s)(1-\tau)(1-s)}{(1-\alpha)[s+\tau(1-s)]}\left(\frac{s+\tau(1-s)}{\delta+n}\right)^{\frac{\alpha}{1-\alpha}} \\ &= (1-s)\left(\frac{s+\tau(1-s)}{\delta+n}\right)^{\frac{\alpha}{1-\alpha}} \frac{\alpha-s-\tau(1-s)}{(1-\alpha)[s+\tau(1-s)]}\end{aligned}$$

Note that $\frac{\partial c_{ss}}{\partial \tau} > 0$ if and only if $\tau < \frac{\alpha-s}{1-s}$. By assumption $s < \alpha$ and τ is positive and small enough, this implies $\frac{\alpha-s}{1-s} > 0$ and $\tau < \frac{\alpha-s}{1-s}$ holds. Thus in long run the new steady state consumption per capita is higher than the level before $\hat{t}$. So this policy increases consumption in long run. The transition path is plotted in the figure below.