Homework - 3
UCLA - 2016
ECON 221C MONETARY ECON III
Liquidity and Financial Friction in Macroeconomics
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1. **From Previous Class.** Please spot typos in the lecture notes corresponding to the class on limited enforcement.

2. **Kiyotaki and Moore.** Consider the version of Kiyotaki and Moore (2012) discussed in class. Maintain the assumption of log preferences. Assume that workers are hand to mouth and entrepreneurs don't supply labor: $\tau_t^{w,k} = 0$, $\tau_t^{e,l} = 0$. Also, specialize to the case where technology is Cobb-Douglas with capital elasticity equal to $1/2$.

   (a) Fix a given amount of aggregate capital K_t. Find the equilibrium amount of hours as a function of K_t. Obtain a function $r^c(K)$ which measure the return per unit of capital.

   (b) As in class, obtain an expression for the investment rate of the economy ι as a function of ϕ_t and q_t for the version of the model where capital is used as collateral.

   (c) As studied in class, derive an expression for the investment rate of the economy ι as a function of ϕ_t and q_t for the version of the model where capital is used as collateral.

   (d) Describe the market clearing condition for capital goods in the cases (b) and (c) —consider sales of used capital in version (c). Obtain an equilibrium equation that allows you to compute the value $q(K, \phi)$. That is, from the market clearing condition, obtain an equilibrium function $q(K, \phi)$ that clears the market respecting the constraints. Notice that since $r^c(K)$ is obtained from an autonomus system, you can write a single implicit equation in $q(K, \phi)$.

   (e) Use the result in (d) to show that equilibrium allocations are the same in both cases (b) and (c).

   (f) Write a simply matlab script that uses the equilibrium equation for q obtained in part (d). For every value of ϕ and a given K, solve for that q. Use it to obtain the solution as in part (b) and use that again to update the value of the capital stock. Report a simulation path for your economy.

3. **Challenge.** You don't have to turn in this equation. I'm just curious cause I haven't seen any closed form solutions to this model. Try setting $\varphi(\iota) = \chi\iota^2/2$. Can you obtain an analytic solution for $q(K, \phi)$ using the formula for the qudratic equation?

4. **Liquidity Premium.** Revert to Epstein-Zin preferences as in the end of class. Assume there's a bond in zero net supply that could be used fully

to relax the borrowing constraint. Use the first order conditions of the certainty equivalent portfolio to derive a "Liquidity Premium". What I mean by liquidity premium is the following. Write :

$$R_t^b = \underbrace{\mathbb{E}\left[\left(\frac{r^c(K) + q_{t+1}\lambda + r^k(\phi, q, K)}{q_t}\right)\right]}_{\substack{\text{Natural} \\ \text{Capital Return}}} + \text{Risk Premium} + \text{Liquidity Premium}$$

The liquidity premium collects the terms that appear only when the collateral constraint is binding. It should have a term that tells us how the collateral constraint binds as the quantity of bonds increases infinitesimally and a term that captures how the collateral constraint is relaxes with more capital —it should depend on ϕ. When the constraint is not active, the liquidity premium should disappear.