Repurchase Options in the Market for Lemons

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Introduction

Motivation

- Modern financial contracts: Repo | Collateralized Debt | Bridge Loans | Factoring | Discounting
  - also early contracts: Pawning | Pignus
  - All have embedded repurchase option
- Why repurchase collateral? Why not simply sell the asset?
  - argue natural response to adverse selection: prevents market unraveling
- Contribution:
  - characterize nature of these contracts in market environment
  - no commitment to a security design ex-ante

Motivating Example

- Investment opportunity w/ 20% return

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Collateral	Value			
Low Quality	\$40			
High Quality	\$80			
	Purchase Price	Repurchase Price	Average Funds Lent	Added Value
Sale	\$40	∞	\$20	\$4
Repo				

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Collateral	Value			
Low Quality	\$40			
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	Purchase Price	Repurchase Price	Average Funds Lent	Added Value
Sale	\$40	∞	\$20	\$4
Repo	\$50	\$60	\$50	\$10

Motivating Example

- What is the nature of market equilibrium?
  - what contracts survive?
  - is the equilibrium efficient?

DETAILS

Environment

- Trade motive: liquidity need + common valuation
- Contract: asset sale + repurchase option
- Modern treatment:
  - Netzer-Scheuer (2014) timing: allow contract withdrawal
  - Miyazaki-Wilson-Spence equilibrium notion

DETAILS

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Results

- Unique pooling equilibrium of ALL assets
  - resolves: adverse selection
  - closed form for any continuous distribution
- Constrained inefficient outcome
  - optimal repo contract = security design solution
  - competition: leads to cream skimming
- When adverse selection under asset sales high, repo dominates outright sales
  - trade-off: increase participation vs. cream skimming

Relation to Literature

Security Design

Demarzo-Duffie (1999), Biais-Mariotti (2005)

- paper: market outcome+no commitment to a security design

Competitive markets with adverse selection

Wilson (1977), Netzer-Scheuer (2014),
Gale (1992,1996), Guerrieri, Shimer, and Wright (2010), Guerrieri and Shimer (2014), Chang (2018)

- focus on asset sales
- paper: richer contract space leads to pooling & improves outcomes

Micro-foundation of repo contracts

Duffie (1996), Dang, Gorton, and Holmström (2010), Monnet and Narajabad (2017), Gottardi, Maurin, and Monnet (2017), Parlatore (2019)

- result from transaction costs (exogenous or endogenous)
- paper: private information

Macro models with private information

Bernanke Gertler (1989), Eisfeldt (2004), Christiano, Motto and Rostagno (2013), Kurlat (2013), Bigio (2015)

- Macro models: e.g. costly-state verification (Townsend, 1979) or Akerlof (1970)
- paper: closed form, portable to macro

The Environment

Two periods: $t = 1, 2$

No discounting

Risk neutral

Agents

Borrowers continuum

- $t = 1$: endowed w/
  - an indivisible (collateral) asset
  - illiquid investment project
- $t = 2$: payouts:
  - asset dividend $\lambda \in \Lambda \equiv \left[\underline{\lambda}, \bar{\lambda}\right] \sim F(\cdot)$
  - project gross payoff $(1 + r) \cdot x$
    - investment $x, r > 0$

Lenders

- indexed by $j \in \mathcal{J}$

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Lenders

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Information asymmetry

- λ borrower private info

Repo Contracts

Specify two prices

$$\boldsymbol{p} = \{p_s, p_r\} \in \left[\underline{\lambda}, \bar{\lambda}\right] \times \left[\underline{\lambda}, \bar{\lambda}\right].$$

- $t = 1$: sales price p_s for asset
- $t = 2$: repurchase price p_r to repossess asset

Repo Contracts

Specify two prices

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- $t = 1$: sales price p_s for asset
- $t = 2$: repurchase price p_r to repossess asset

Borrower repurchase option

- borrower can default
- lender commits to return asset if paid
- outright asset sales: special case ($p_r = \bar{\lambda}$)

Repo Market

Stage 1: Each lender offers a contract

- The set of offered contracts, observed by all

$$\mathbb{P}_0 = \left\{ \boldsymbol{p}^j : \forall j \in \mathcal{J} \right\}$$

Stage 2: Contract withdrawal

- Remaining contracts:

$$\mathbb{P} = \left\{ \boldsymbol{p}^j \in \mathbb{P}_0 : I^j = 1, \forall j \in \mathcal{J} \right\}$$

where $I^j = 1$: not withdrawn

Stage 3:

- Borrowers: choose $\boldsymbol{p}$ among $\mathbb{P}$ or opt out

Agents' Problems

Borrower

$$\max\{0, v(\lambda)\}$$

where

$$v(\lambda) = \max_{p \in \mathbb{P}} \left\{ (1+r)\, p_s - \underbrace{\min\{\lambda, p_r\}}_{\text{default?}} \right\}$$

Agents' Problems

Borrower

$$\max\left\{0, v\left(\lambda\right)\right\}$$

where

$$v\left(\lambda\right) = \max_{\boldsymbol{p}\in\mathbb{P}}\left\{\left(1+r\right)p_s - \underbrace{\min\left\{\lambda, p_r\right\}}_{\text{default?}}\right\}$$

Lender

$$\Pi^j\left(\boldsymbol{p}^j, \mathbb{P}^{-j}, \mathbb{P}_0^{-j}\right) = \max\left\{\int \min\left\{\lambda, p_r^j\right\} \underbrace{d\Gamma\left(\lambda | \boldsymbol{p}^j, \mathbb{P}^{-j}\cup\boldsymbol{p}^j\right)}_{\text{distribution of quality}} - p_s^j, 0\right\}$$

where

$$\mathbb{P}^{-j} = \left\{\boldsymbol{p}^k \in \mathbb{P}_0 : I^k = 1, \forall k \in \mathcal{J}/j\right\}$$

Optimal Borrower Strategy

Lemma 1. Full Participation and Partial Default

1. [*Full participation*] All borrowers sign a repo contract
2. [*Default threshold*] $\exists!$ threshold $\lambda_d \leq \bar{\lambda}$ s.t. all lower quality assets default

Borrower Contract Choice

Two contracts (wlog):

- Highest sales price & highest non-default value

$$\boldsymbol{p}^d \equiv \underset{\boldsymbol{p}\in\mathbb{P}}{\operatorname{argmax}}\, p_s, \qquad \boldsymbol{p}^n \equiv \underset{\boldsymbol{p}\in\mathbb{P}}{\operatorname{argmax}} \left\{(1+r)\, p_s - p_r\right\}$$

Lemma 2. Borrower Contract Choice

Defaulters:

$$P(\lambda) = \boldsymbol{p}^d \text{ and } v(\lambda) > \bar{v},\ \forall\lambda \in [\underline{\lambda}, \lambda_d)$$

Non-defaulters:

$$P(\lambda) = \boldsymbol{p}^n \text{ and } v(\lambda) = \bar{v},\ \forall\lambda \in \left[\lambda_d, \bar{\lambda}\right]$$

Pooling Equilibrium

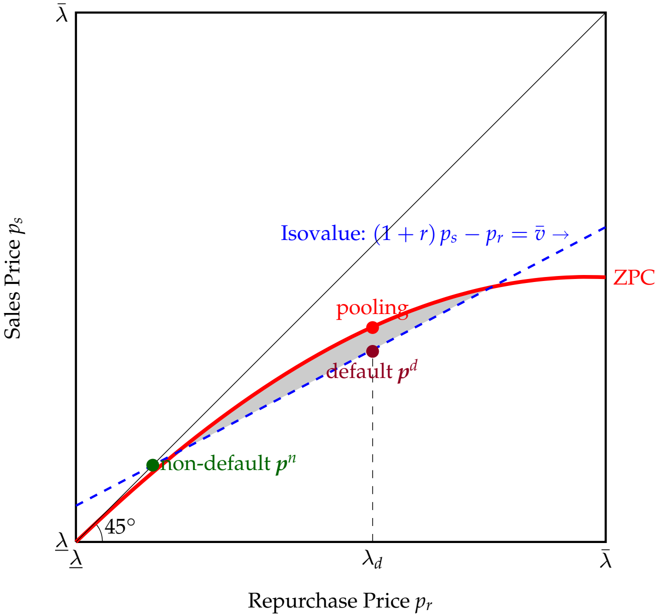
Proposition 1. Pooling

Equilibrium features a pooling contract $\boldsymbol{p}^n = \boldsymbol{p}^d = \boldsymbol{p}$ with (p_s, p_r):

1. [*Repurchase price*]
$$p_r = \lambda_d$$

2. [*ZPC*]
$$p_s = \mathbb{E}\left[\min\left\{\lambda, p_r\right\}\right]$$

Pooling Equilibrium



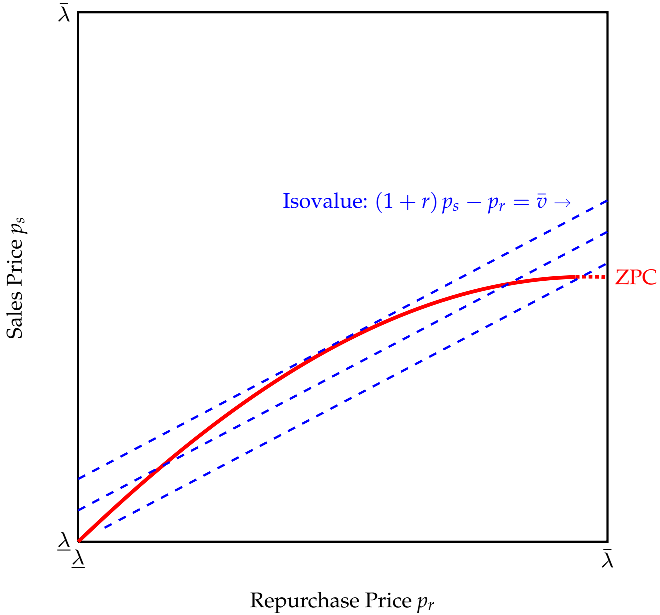
Unique Equilibrium

Proposition 2. Uniqueness

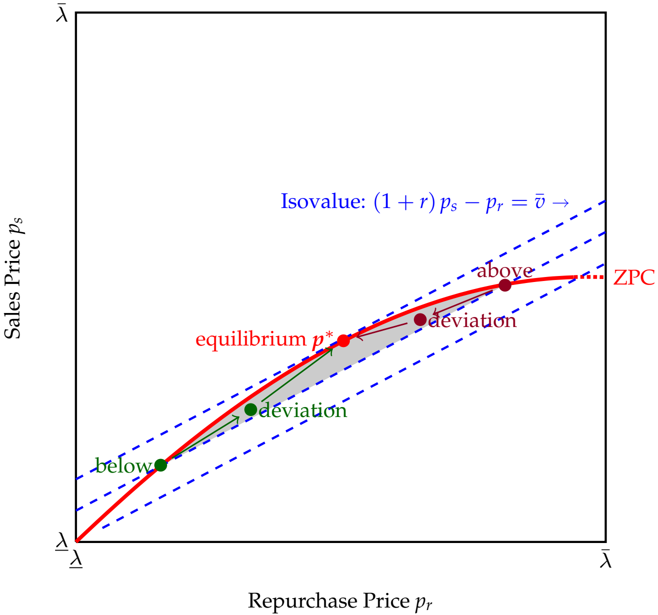
Unique equilibrium: a **single** zero-profit pooling contract

$$\boldsymbol{p}^* = \underset{p_s = \mathbb{E}[\min\{\lambda, p_r\}]}{\operatorname{argmax}} \{(1+r)\, p_s - p_r\}$$

Unique Equilibrium



Unique Equilibrium



Analytic Solution

Equilibrium Contract p^*

Repurchase price:

$$p_r^* = F^{-1}\left(\frac{r}{1+r}\right)$$

Sales price:

$$p_s^* = \mathbb{E}\left[\min\left\{\lambda, F^{-1}\left(\frac{r}{1+r}\right)\right\}\right]$$

Default rate:

$$d = \frac{r}{1+r}$$

Optimal Repo Contract Design

Mechanism Design:

$$\max_{\{P(\cdot),\lambda^p\}} \int_{\underline{\lambda}}^{\lambda^p} \left((1+r)\,P_s(\lambda) - \min\{\lambda, P_r(\lambda)\}\right) dF(\lambda)$$

s.t.

1) Incentive Compatibility
2) Participation Constraint
3) Budget Balance

Constrained Efficiency: Solution

Condition 1. Heterogeneity.

$(1+r)\,\mathbb{E}\left[\lambda\right] < \bar{\lambda}$

Proposition 4. Constrained Efficiency

Under Condition 1, the optimal contract is a full-participation pooling contract:

$$\boldsymbol{p}^{p} \in \underset{p_s = \mathbb{E}[\min\{\lambda, p_r\}]}{\operatorname{argmax}} \; p_s$$

st:

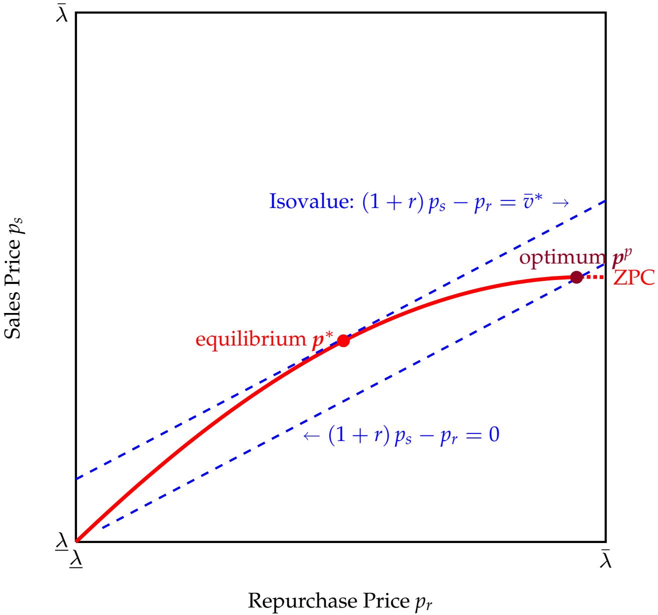
$$\bar{v} = (1+r)\,p_s - p_r \geq 0$$

- Binding participation & max cross-subsidization:

$$\bar{v}^{p} = (1+r)\,p_s^{p} - p_r^{p} = 0$$

- Optimal security design: Demarzo-Duffie (1999) & Biais-Mariotti (2005)

Optimal Repo Contract



Source of Inefficiency

Market solution:

$$\boldsymbol{p}^* = \underset{p_s = \mathbb{E}[\min\{\lambda, p_r\}]}{\operatorname{argmax}} \{(1+r)\, p_s - p_r\}$$

Planner solution:

$$\boldsymbol{p}^p \in \underset{p_s = \mathbb{E}[\min\{\lambda, p_r\}]}{\operatorname{argmax}} \quad p_s$$

Source of inefficiency:

- Lack of separation: No
- Adverse selection: No
- Cream skimming: Yes

Repo vs. Sales: Efficiency Comparison

Repos vs. Sales: tradeoff adverse selection vs. cream skimming

Statistics

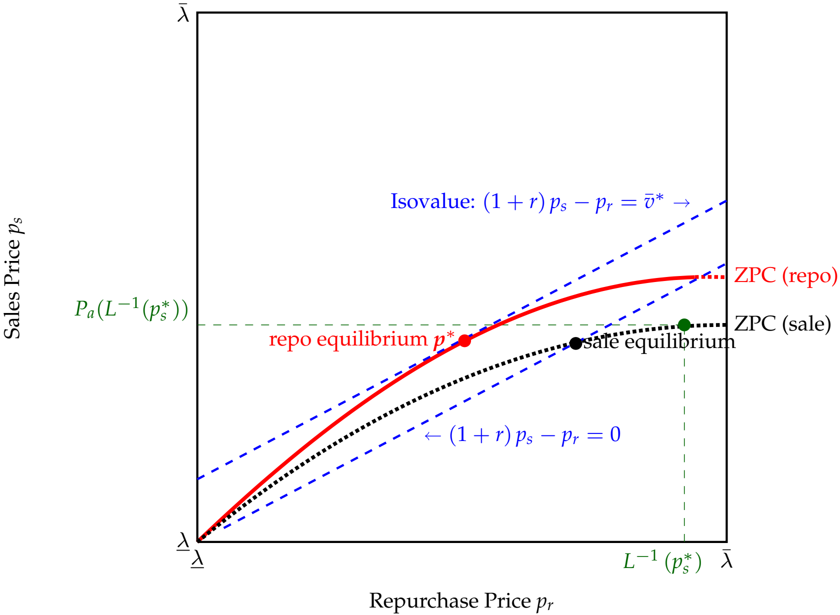
$$Z_a(\lambda) \equiv \mathbb{E}\left[\tilde{\lambda} \,\middle|\, \tilde{\lambda} \leq \lambda\right] \text{ and } L_a(\lambda) \equiv \mathbb{E}\left[\tilde{\lambda} \,\middle|\, \tilde{\lambda} \leq \lambda\right] F(\lambda), \; \forall \lambda \in \Lambda$$

Proposition 5. Sufficient Statistics

- Repo dominates sales iff:

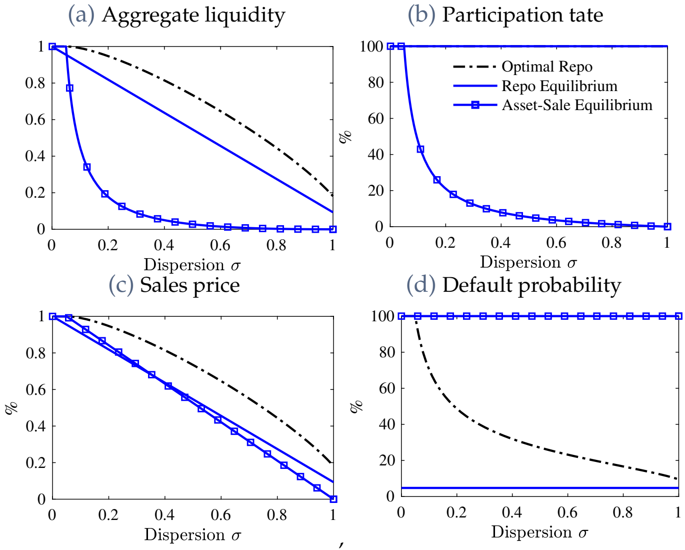
$$(1+r)\, Z_a\left(L_a^{-1}(p_s^*)\right) < L_a^{-1}(p_s^*)$$

Repo vs. Sales: Efficiency Comparison

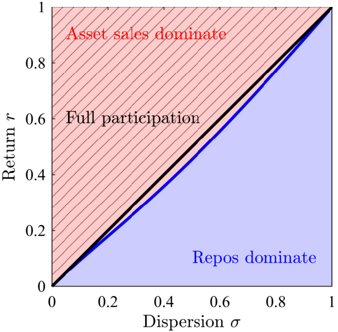


UNIFORM DISTRIBUTION EXAMPLE

Example. $\lambda \sim \mathrm{U}[1-\sigma, 1+\sigma], r = 5\%$



Uniform Distribution Example



Extensions & Variations

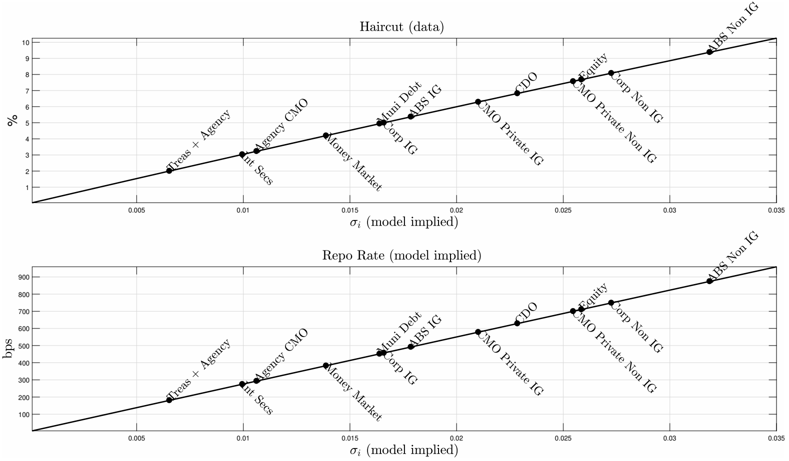
- Lenders offer multiple contracts?
  - immaterial
- Tax on repos
  - immaterial with budget balance
- Lender's lack of commitment
  - effect on participation
- Repo under competitive search (Guerrieri, Shimer, and Wright (2010))
  - obtain unique pooling equilibrium
  - enriching contract space improves outcomes
  - repo always dominates asset sales

Evidence from Repo Markets

- Big haircut movements (Gorton and Metrick)
  - no corresponding increase in risk

- What Drives Repo Haircuts? by Julliard, Liu, Seyedan, Todorov, Yuan
  - measure of greater uncertainty | information
  - collateral quality, maturity

Haircuts in the Data and Model Fit



CONCLUSION

Summary

- Repos or collateralized debt, widely used in financial markets. Why?
- Natural outcome in markets with private information
- Puzzle: large haircuts in comparison with default
  - consistent with the equilibrium features here